College and Career Readiness Standards for Reading, Writing, and Speaking and Listening

Draft for Review and Comment

September 21, 2009

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Table of Contents

Introduction
Core Standards for Reading, Writing, and Speaking and Listening i
Student Practices in Reading, Writing, and Speaking and Listening iii
Introductory Evidence Statement for Reading, Writing, and Speaking and Listening Standards v
How to Read the Document vii

Core Standards for Reading Informational and Literary Texts
Standards for the Range and Content of Student Reading 1-A
Standards for Student Performance 1-B

Core Standards for Writing
Standards for the Range and Content of Student Writing 2-A
Standards for Student Performance 2-B

Core Standards for Speaking and Listening
Standards for the Range and Content of Student Speaking and Listening 3-A
Standards for Student Performance 3-B

Applications of the Core
Application of the Core: Research 4-A
Application of the Core: Media 4-B

Illustrative Texts: Exemplars of Reading Text Complexity
Introduction 5
Illustrative Texts with Annotations 6

~~**Illustrative Student Writing Samples**~~ **- Coming in next draft**
~~Introduction~~
~~Student Writing Samples with Annotations~~

Sample of Works Consulted 31

Core Standards for Reading, Writing, and Speaking and Listening

The Core Standards identify essential college- and career-ready skills and knowledge in reading, writing, and speaking and listening across the disciplines. While the English language arts classroom has often been seen as the proper site for literacy instruction, this document acknowledges that the responsibility for teaching such skills must also extend to the other content areas. Teachers in the social and natural sciences, the humanities, and mathematics need to use their content area expertise to help students acquire the discipline-specific skills necessary to comprehend challenging texts and develop deep knowledge in those fields. At the same time, English language arts teachers not only must engage their students in a rich array of literature but also must help develop their students' ability to read complex works of nonfiction independently.

What is taught is just as important as how it is taught; the Core Standards should be accompanied by a comprehensive, content-rich curriculum. While this document defines the outcomes all students need to reach to be college and career ready, many important decisions about curriculum will necessarily be left to states, districts, schools, teachers, professional organizations, and parents. For example, while the standards require that students read texts of sufficient complexity, quality, and range, this document does not contain a required reading list. If states and districts choose to develop one, they should look at the Reading exemplars provided here to get a sense of the level of complexity students must be able to handle independently when they read. Educators can also model their efforts on reading lists from around the nation and the world as long as the texts ultimately included meet the range and content standards in this document.

Standards today must ready students for competition and collaboration in a global, media-saturated environment. Colleges and universities have become international meetinghouses where people from across the globe learn with and from one another. At the same time, business today is truly a worldwide enterprise. Media-related technology helps shape what goes on in both college and the workplace; indeed, it has in some important ways reshaped the very nature of communication. Students who meet the Core Standards will have the reading, writing, speaking, and listening skills to flourish in the diverse, rapidly changing environments of college and careers.

Although reading, writing, and speaking and listening are articulated separately in the standards that follow, these divisions are made for the sake of clarity and manageability. In reality, the processes of communication are tightly interrelated and often reciprocal. The act of reading can no more be separated from the written word than the act of listening can be from the spoken word. When reading, students demonstrate their comprehension most commonly through a spoken or written interpretation of the text. As students solve problems, share insights, and build the

knowledge they need for college and career success, they draw simultaneously on their capacities to read, write, speak, and listen.

Student Practices in Reading, Writing, and Speaking and Listening

The following practices in reading, writing, and speaking and listening undergird and help unify the rest of the standards document. They are the "premises"—broad statements about the nature of college and career readiness in reading, writing, and speaking and listening—that underlie the individual standards statements and cut across the various sections of the document. Every idea introduced here is subsequently represented in one or more places within the larger document.

* * *

Students who are college and career ready exhibit the following capacities in their reading, writing, and speaking and listening:

1. *They demonstrate independence as readers, writers, speakers, and listeners.*

   Students can, without significant scaffolding or support, comprehend and evaluate complex text across a range of types and disciplines, and they can construct effective arguments and clearly convey intricate or multifaceted information. Likewise, students are independently able to discern a speaker's key points as well as ask questions and articulate their own ideas.

2. *They build strong content knowledge.*

   Students build a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They demonstrate their ability to become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and the specific in-depth expertise needed to comprehend subject matter and solve problems in different fields. They refine their knowledge and share it through substantive writing and speaking.

3. *They respond to the varying demands of audience, task, purpose, and discipline.*

   Students consider their reading, writing, and speaking and listening in relation to the contextual factors of audience, task, purpose, and discipline. They appreciate nuances, such as how the composition and familiarity of the audience should affect tone. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in the natural sciences).

4. *They comprehend as well as critique.*

   Students are engaged and open-minded—but skeptical—readers and listeners. They work diligently to understand precisely what an author or speaker is

saying, but they also question an author's or speaker's assumptions and assess the veracity of claims.

5. *They privilege evidence.*

   Students cite specific textual evidence when offering an oral or written interpretation of a piece of writing. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

6. *They care about precision.*

   Students are mindful of the impact of specific words and details, and they consider what would be achieved by different choices. Students pay especially close attention when precision matters most, such as in the case of reviewing significant data, making important distinctions, or analyzing a key moment in the action of a play or novel.

7. *They craft and look for structure.*

   Students attend to structure when organizing their own writing and speaking as well as when seeking to understand the work of others. They understand and make use of the ways of presenting information typical of different disciplines. They observe, for example, how authors of literary works craft the structure to unfold events and depict the setting.

8. *They use technology strategically and capably.*

   Students employ technology thoughtfully to enhance their reading, writing, speaking, and listening. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

Introductory Evidence Statement for Reading, Writing, and Speaking and Listening Standards

To develop college- and career-ready standards for Reading, Writing, and Speaking and Listening that are rigorous, relevant, and internationally benchmarked, the work group consulted evidence from a wide array of sources. These included standards documents from high-performing states and nations; student performance data (including assessment scores and college grades); academic research; frameworks for assessments, such as NAEP; and results of surveys of postsecondary instructors and employers regarding what is most important for college and career readiness.

The evidence strongly suggests that similar reading, writing, speaking, and listening skills are necessary for success in both college and the workplace. A review of the standards of high-performing nations also suggests that many of these skills are already required in secondary schools internationally. The work group has endeavored to articulate these skills in the Core Standards, focusing educators, students, parents, and resources on what matters most.

Given that a set of standards cannot be simplistically "derived" from any body of evidence, the work group sometimes relied on reasoned judgment to interpret where the evidence was most compelling. For example, there is not a consensus among college faculty about the need for incoming students to be able to comprehend graphs, charts, and tables and to integrate information in these data displays with the information in the accompanying text. Although some evidence suggests that this skill is critical in the workplace and in some entry-level courses, college faculties from the various disciplines disagree on its value (with science and economics faculty rating it more highly than English and humanities professors do). The work group ultimately included a standard on the integration of text and data because the preponderance of the evidence suggests the skill's importance in meeting the demands of the twenty-first-century workplace and some college classrooms.

In most cases, the evidence is clearer. In writing, for example, there is unequivocal value placed on the logical progression of ideas. The expectation that high school graduates will be able to produce writing that is logical and coherent is found throughout the standards of top-performing countries and states. This ability is also valued highly by college faculty and employers. In response to such clear evidence, the work group included Writing student performance standard #5: "Create a logical progression of ideas or events, and convey the relationships among them."

A bibliography of some of the sources the work group drew upon most is included at the end of this document. The reader should also refer to the Core Standards Web site (http://www.corestandards.org), which contains a list of standards linked to relevant sources of evidence.

Finally, while the standards reflect the best evidence available to date, the decisions the work group made are necessarily provisional. The core should be reexamined periodically as additional research on college and career readiness emerges. Indeed, this document may serve as an agenda for such research.

How to Read the Document

This document is divided into three main sections: strands, applications, and supporting materials.

Strands
There are three *strands*: Reading, Writing, and Speaking and Listening. Although each strand is presented discretely for ease of understanding, the document should be considered a coherent whole.

The three strands are each in turn divided into two sections: S*tandards for Range and Content* and S*tandards for Student Performance*.

> **Standards for Range and Content**
> The Standards for Range and Content in each strand describe the contexts in which college- and career-ready students must be able to read, write, speak, and listen. Rather than merely supplement or illustrate the numbered list of Standards for Student Performance, the Standards for Range and Content are themselves required and carry equal force.
>
> **Standards for Student Performance**
> The Standards for Student Performance in each strand enumerate the essential skills and understandings that students who are college and career ready in reading, writing, speaking, and listening must have no later than the end of high school.

Applications
The clearest examples of the integrated nature of communication are the *Applications of the Core* for Research and Media. The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks, such as research and media. Rather than having an additional set of standards that would largely duplicate those already in Reading, Writing, and Speaking and Listening, the document includes the Research and Media applications that draw upon standards already in those strands. This both reaffirms the centrality of the core processes of reading, writing, speaking, and listening and shows how those processes can be combined and extended to describe key communicative acts in the classroom and workplace.

In the Research and Media applications, specific Reading, Writing, and Speaking and Listening standards are identified with a letter or letters corresponding to the relevant strand (R for Reading, W for Writing, and S&L for Speaking and Listening) and a number or letter corresponding to the statement within that strand. For example, R-14 refers to the fourteenth statement in the Standards for Student

Performance in Reading, and W-A refers to the first statement of the Standards for Range and Content in Writing.

Supporting Materials: Reading and Writing Exemplars
Reading and Writing exemplars, and their accompanying annotations, are used to lend further specificity to the standards.

> **Reading Exemplars**
> The Reading exemplars, representing a range of subject areas, time periods, cultures, and formats, illustrate the level of text complexity students ready for college and careers must be able to handle on their own. The exemplars are mostly excerpts or representations of larger works. To be truly college and career ready, students must be able to handle full texts—poems, short stories, novels, technical manuals, research reports, and the like. Annotations accompanying the exemplars explain how each text meets the criterion of high text complexity. The annotations also provide brief performance examples that further clarify the meaning and application of the standards.
>
> ~~**Writing Exemplars**~~ **- Coming in the next draft**
> ~~The Writing exemplars are authentic samples of student writing created across the nation under a variety of conditions and for a variety of purposes and audiences. Annotations accompanying the exemplars indicate how these samples meet the Standards for Student Performance in Writing.~~

Core Standards for Reading Informational and Literary Texts

Standards for the Range and Content of Student Reading

A. **Complexity:** A crucial factor in readiness for college and careers is students' ability to comprehend complex texts independently. In college and careers, students will need to read texts characterized by demanding vocabulary, subtle relationships among ideas or characters, a nuanced rhetorical style and tone, and elaborate structures or formats. These challenging texts require the reader's close attention and often demand rereading in order to be fully understood.

B. **Quality:** The literary and informational texts chosen for study should be rich in content and in a variety of disciplines. All students should have access to and grapple with works of exceptional craft and thought both for the insights those works offer and as models for students' own thinking and writing. These texts should include classic works that have broad resonance and are alluded to and quoted often, such as influential political documents, foundational literary works, and seminal historical and scientific texts. Texts should also be selected from among the best contemporary fiction and nonfiction and from a diverse range of authors and perspectives.

C. **Vocabulary:** To be college and career ready, students must encounter and master a rich vocabulary. Complex texts often use challenging words, phrases, and terms that students typically do not encounter in their daily lives. Specific disciplines and careers have vocabularies of their own. Attentive reading of sophisticated works in a wide range of fields, combined with close attention to vocabulary, is essential to building comprehension and knowledge.

D. **Range:** Students must be able to read a variety of literature, informational texts, and multimedia sources in order to gain the knowledge base they need for college and career readiness.

Literature: Literature enables students to access through imagination a wide range of experiences. By immersing themselves in literature, students enlarge their experiences and deepen their understanding of their own and other cultures. Careful reading of literature entails attentiveness to craft and details of design, which has broad value for students' work in college and career environments.

Informational Text: Because most college and workplace reading is nonfiction, students need to hone their ability to acquire knowledge from informational texts. Workplace and discipline-specific reading will often require students to demonstrate persistence as they encounter a large amount of unfamiliar and often technical vocabulary and concepts. Students must demonstrate facility with the features of texts particular to a variety of disciplines, such as history, science, and mathematics.

Multimedia Sources: Students must be able to integrate what they learn from reading text with what they learn from audio, video, and other digital media. Many of the same critical issues that students face when reading traditional printed texts will arise as they seek to comprehend multimedia, such as determining where the author has chosen to focus, evaluating evidence, and comparing different accounts of similar subjects.

E. **Quantity*:*** Students must have the capacity to handle independently the quantity of reading material, both in print and online, required in college and workforce training. Studies show that the amount of reading students face in high school is often far lower than that required for typical first-year college courses. Students need to be able to perform a close reading of a much higher volume of texts and to sort efficiently through large amounts of print and online information in search of specific facts or ideas.

Note: *The essential role of independence in college and career readiness:* The significant scaffolding that often accompanies reading in high school usually disappears in college and workforce training environments. Students must therefore have developed their ability to read texts of sufficient complexity, quality, and range on their own. To become independent, students must encounter unfamiliar texts presented without supporting materials.

Core Standards for Reading Informational and Literary Texts

Standards for Student Performance

1. Determine both what the text says explicitly and what can be inferred logically from the text.
2. Support or challenge assertions about the text by citing evidence in the text explicitly and accurately.
3. Discern the most important ideas, events, or information, and summarize them accurately and concisely.
4. Delineate the main ideas or themes in the text and the details that elaborate and support them.
5. Determine when, where, and why events unfold in the text, and explain how they relate to one another.
6. Analyze the traits, motivations, and thoughts of individuals in fiction and nonfiction based on how they are described, what they say and do, and how they interact.
7. Determine what is meant by words and phrases in context, including connotative meanings and figurative language.
8. Analyze how specific word choices shape the meaning and tone of the text.
9. Analyze how the text's organizational structure presents the argument, explanation, or narrative.
10. Analyze how specific details and larger portions of the text contribute to the meaning of the text.
11. Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension.
12. Extract key information efficiently in print and online using text features and search techniques.
13. Ascertain the origin, credibility, and accuracy of print and online sources.
14. Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient.
15. Analyze how two or more texts with different styles, points of view, or arguments address similar topics or themes.
16. Draw upon relevant prior knowledge to enhance comprehension, and note when the text expands on or challenges that knowledge.
17. Apply knowledge and concepts gained through reading to build a more coherent understanding of a subject, inform reading of additional texts, and solve problems.
18. Demonstrate facility with the specific reading demands of texts drawn from different disciplines, including history, literature, science, and mathematics.

Note: *These Standards for Student Performance, as is the case for every strand, must be demonstrated across the range and content from the preceding page.* They are meant to apply to fiction and nonfiction. For example:

- "Determine when, where, and why events unfold" applies to plot and setting in literature as well as the sequence of a scientific procedure.
- "Analyze the traits, motivations, and thoughts of individuals" applies to studying characters in fiction and figures in historical texts.

Core Standards for Writing

Standards for the Range and Content of Student Writing

A. **Purpose:**

Make an Argument: While many high school students have experience presenting their opinions, they need to be able to make arguments supported by evidence in order to be ready for careers and college. Students must be able to frame the debate over a claim, present the reasoning and evidence for the argument, and acknowledge and address its limitations. In some cases, students will make arguments to gain entry to college or to obtain a job, laying out their qualifications or experience. In college, students might defend an interpretation of a work of literature or of history; in the workplace, employees might write to recommend a course of action.

Inform or Explain: In college and in workforce training, writing is a key means for students to show what they know and to share what they have seen. Writing to inform or explain often requires students to integrate complex information from multiple sources in a lucid fashion. Explanations can take the form of laying out facts about a new technology or documenting findings from historical research; well-crafted explanations often make fresh connections and express ideas creatively.

B. **Audience:** Students must adapt their writing so that it is appropriate to the audience by choosing words, information, structures, and formats that conform to the conventions of the discipline in which they are writing. The form and use of evidence in literary analysis, for example, are likely to be quite different from those in geology or business. Students must also be able to consider their audience's background knowledge and potential objections to an argument.

C. **Situation:**

On-demand Writing: Students must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline. College and career readiness requires that students be able to write effectively to a prompt on an exam or respond quickly yet thoughtfully to a supervisor's urgent request for information.

Writing over Time: Students must be able to revisit and make improvements to a piece of their writing over multiple drafts when circumstances encourage or require it. To improve writing through revision, students must be capable of distinguishing good changes from ones that would weaken the writing.

D. **Technology and Collaboration:** Technology offers students powerful tools for producing, editing, and distributing writing as well as for collaboration. Especially in the workplace, writers often use technology to produce documents and to provide feedback.

E. **Quantity:** The evidence is clear that, in order to become better writers, students must devote significant time to producing writing. Students must practice writing several analytical pieces each term if they are to achieve the deep analysis and interpretation of content expected for college and careers.

Note on narrative writing:

Narrative writing is an important mode of writing; it is also a component of making an argument and writing to inform or explain. Telling an interesting story effectively or providing an accurate account of a historical incident requires the skillful use of narrative techniques. Narrative writing requires that students present vivid, relevant details to situate events in a time and place and also craft a structure that lends a larger shape and significance to those details. As an easily grasped and widely used way to share information and ideas with others, narrative writing is a principal stepping-stone to writing forms directly relevant to college and career readiness.

Core Standards for Writing

Standards for Student Performance

1. Establish and refine a topic or thesis that addresses the specific task and audience.
2. Gather the information needed to build an argument, provide an explanation, or address a research question.
3. Sustain focus on a specific topic or argument.
4. Support and illustrate arguments and explanations with relevant details, examples, and evidence.
5. Create a logical progression of ideas or events, and convey the relationships among them.
6. Choose words and phrases to express ideas precisely and concisely.
7. Use varied sentence structures to engage the reader and achieve cohesion between sentences.
8. Develop and maintain a style and tone appropriate to the task, purpose, and audience.
9. Demonstrate command of the conventions of standard written English, including grammar, usage, and mechanics.
10. Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism.
11. Assess the quality of one's own writing, and, when necessary, strengthen it through revision.
12. Use technology as a tool to produce, edit, and distribute writing.

When **writing to inform or explain**, students must also do the following:

13. Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information.
14. Convey complex information clearly and coherently to the audience through purposeful selection and organization of content.
15. Demonstrate understanding of content by reporting facts accurately and anticipating reader misconceptions.

When **writing arguments**, students must also do the following:

16. Establish a substantive claim, distinguishing it from alternate or opposing claims.
17. Link claims and evidence with clear reasons, and ensure that the evidence is relevant and sufficient to support the claims.
18. Acknowledge competing arguments or information, defending or qualifying the initial claim as appropriate.

Note: "The conventions of standard written English" encompass a range of commonly accepted language practices designed to make writing clear and widely understood. When formal writing contains errors in grammar, usage, and mechanics, its meaning is obscured, its message is too easily dismissed, and its author is often judged negatively. Proper sentence structure, correct verb formation, careful use of verb tense, clear subject-verb and pronoun-antecedent agreement, conventional usage, and appropriate punctuation are of particular importance to formal writing.

Core Standards for Speaking and Listening

Standards for the Range and Content of Student Speaking and Listening

A. **Group and One-to-One Situations:** Students are expected to be able to speak and listen effectively in both groups and one-to-one. Success in credit-bearing college coursework, whether in the humanities, mathematics, or the sciences, depends heavily on being able to take in and respond to the concepts and information conveyed in lectures and class discussions. Success in the workplace is similarly dependent on listening attentively to colleagues and customers and expressing ideas clearly and persuasively.

   These speaking and listening skills may need to be applied differently in different settings. The immediate communication between two people might be replaced by formal turn taking in large-group discussions. When working in classroom or workplace teams, students should be able to ask questions that initiate thoughtful discussions, gain the floor in respectful ways, and build on the contributions of others to complete tasks or reach consensus.

B. **Varied Disciplinary Content:** Students must adapt their speaking and listening to a range of disciplines to communicate effectively. Each academic discipline and industry has its own vocabulary and conventions; for instance, evidence is handled and discussed differently in literary analysis than in history or medicine or the sciences. College- and career-ready students must develop a foundation of disciplinary knowledge and conventions in order not only to comprehend the complexity of information and ideas but also to present and explain them.

C. **Multimedia Comprehension:** New technologies expand the role that speaking and listening skills will play in acquiring and sharing knowledge. Students will need to view and listen to diverse media to gain knowledge and also must integrate this information with what they learn through reading text online as well as in print. When speaking, students can draw on media to illustrate their points, make data and evidence vivid, and engage their audience. Multimedia accelerates the speed at which connections between reading, writing, speaking, and listening can be made, requiring students to be ready to use these skills nearly simultaneously.

Core Standards for Speaking and Listening

Standards for Student Performance

1. Select and use a format, organization, and style appropriate to the topic, purpose, and audience.
2. Present information, findings, and supporting evidence clearly and concisely.
3. Make strategic use of multimedia elements and visual displays of data to gain audience attention and enhance understanding.
4. Demonstrate command of formal Standard English when appropriate to task and audience.
5. Listen to complex information, and discern the main ideas, the significant details, and the relationships among them.
6. Follow the progression of the speaker's message, and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric.
7. Ask relevant questions to clarify points and challenge ideas.
8. Respond constructively to advance a discussion and build on the input of others.

Note: "Style appropriate to the topic, purpose, and audience" includes word choice specific to the demands of the discipline as well as delivery techniques such as gestures and eye contact that contribute to effective message delivery.

"Evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric" includes distinguishing facts from opinions and determining whether the speaker is biased and evidence has been distorted.

Application of the Core: Research

The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks such as research. This section shows how standards in the core incorporate the skills of research.

To be college and career ready, students must engage in research and present their findings in writing and orally, in print and online. The ability to conduct research independently and effectively plays a fundamental role in gaining knowledge and insight in college and the workplace.

Research as described here is not limited to the formal, extended research paper nor simply to gathering information from books; rather, research encompasses a flexible yet systematic approach to resolving questions and investigating issues through the careful collection, analysis, synthesis, and presentation of information from a wide range of print and digital sources, such as historical archives and online interviews. With well-developed research skills, students have the tools to engage in sustained inquiry as well as the sort of short, focused research projects that typify many assignments in college and the workplace.

Research in the digital age offers new possibilities as well as new or heightened challenges. While the Internet provides ready access to unprecedented amounts of primary and secondary source material (such as oral histories, historical documents, maps, and scientific reports), students sorting through this wealth of data must be skilled at and vigilant in determining the origin and credibility of these sources.

The following Core Standards pertain to elements of the research process and particular research skills required for college and career readiness:

Formulate research questions:
- Establish and refine a topic or thesis that addresses the specific task and audience. (W-1)
- Establish a substantive claim, distinguishing it from alternate or opposing claims. (W-16)

Gather and evaluate relevant information from a range of sources:
- Gather the information needed to build an argument, provide an explanation or address a research question. (W-2)
- Extract key information efficiently in print and online using text features and search techniques. (R-12)
- Ascertain the origin, credibility, and accuracy of print and online sources. (R-13)
- Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient. (R-14)
- Follow the progression of the speaker's message and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric. (S&L-6)

Analyze research sources:
- Delineate the main ideas or themes in the text and the details that elaborate and support them. (R-4)
- Listen to complex information and discern the main ideas, the significant details, and the relationships among them. (S&L-5)
- Discern the most important ideas, events, or information and summarize them accurately and concisely. (R-3)
- Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension. (R-11)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information. (W-13)
- Analyze how two or more texts with different styles, points of view, or arguments address similar topics or themes. (R-15)
- Acknowledge competing arguments or information, defending or qualifying the initial claim as appropriate. (W-18)

Report findings:
- Link claims and evidence with clear reasons and ensure that the evidence is relevant and sufficient to support the claims. (W-17)
- Convey complex information clearly and coherently to the audience through purposeful selection and organization of the content. (W-14)
- Demonstrate understanding of the content by reporting the facts accurately and anticipating reader misconceptions. (W-15)
- Present information, findings, and supporting evidence, clearly and concisely. (S&L-2)
- Support and illustrate arguments and explanations with relevant details, examples, and evidence. (W-4)
- Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism. (W-10)

Application of the Core: Media

The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks such as media analysis and creation. This section shows how standards in the core apply to media.

Rapidly evolving technologies are powerful tools—but only for those who have the skills to put them to work. As the capability of the technology grows, students' command of these skills must only increase.

At the core of media mastery are the same fundamental capacities as are required offline in traditional print forms: an ability to access, understand, and evaluate complex materials and messages and to produce clear, effective communications. Media mastery does, however, call upon students to apply these core skills in new ways and contexts. Media enable students to communicate quickly with a large, often unknown, and broadly diverse audience. Whereas in the past, students may have had days or weeks to digest new information and formulate a response, the online environment pushes students to exercise judgment and present their responses in a matter of minutes.

Speed is not the only new factor. In the electronic world, reading, writing, speaking, and listening are uniquely intertwined. Multimedia forms force students to engage with constantly changing combinations of elements, such as graphics, images, hyperlinks, and embedded video and audio. The technology itself is changing quickly, creating new urgency for adaptation and flexibility on the part of students.

The following Core Standards describe the particular reading, writing, speaking, and listening skills that students will need in order to use media effectively in college and careers:

Standards for Range and Content drawn from each strand

Multimedia Sources: Students must be able to integrate what they learn from reading text with what they learn from audio, video, and other digital media. Many of the same critical issues that students face when reading traditional printed texts will arise as they seek to comprehend multimedia, such as determining where the author has chosen to focus, evaluating evidence, and comparing different accounts of similar subjects. [R-D]

Technology and Collaboration: Technology offers students powerful tools for producing, editing, and distributing writing as well as for collaboration. Especially in the workplace, writers often use technology to produce documents and to provide feedback. [W-D]

Multimedia Comprehension: New technologies expand the role that speaking and listening skills will play in acquiring and sharing knowledge. Students will need to view and listen to diverse media to gain knowledge and integrate this information with what they learn through reading text online as well as in print. When speaking, students can draw on media to illustrate their points, make data and evidence vivid, and engage their audiences. Multimedia accelerates the speed at which connections between reading, writing, and speaking and listening can be made, requiring students to be ready to use these skills nearly simultaneously. [S&L-C]

Standards for Student Performance drawn from each strand

Gather information from a wide array of electronic sources and multimedia:
- Extract key information efficiently in print and online using text features and search techniques. (R-12)
- Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension. (R-11)
- Listen to complex information and discern the main ideas, the significant details, and the relationships among them. (S&L-5)

Evaluate information from digital media:
- Ascertain the origin, credibility, and accuracy of print and online sources. (R-13)
- Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient. (R-14)
- Follow the progression of the speaker's message and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric. (S&L-6)

Create and distribute media communications:
- Use technology as a tool to produce, edit, and distribute writing. (W-12)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information. (W-13)
- Make strategic use of multimedia elements and visual displays of data to gain audience attention and enhance understanding. (S&L-3)

Illustrative Texts

Exemplars of Reading Text Complexity

As described in the Standards for the Range and Content of Student Reading, college- and career-ready students must be able to read texts of sufficient complexity on their own. Studies show that many students who are unable to read sufficiently challenging texts independently by the end of high school struggle with the reading demands of college; many twenty-first-century careers likewise demand that people be able to obtain, search through, and comprehend large amounts of often technical information.

To develop that ability, students should engage with high-quality texts that provide strong models of thinking and writing, that challenge them intellectually, and that introduce them to rich content, sophisticated vocabulary, and examples of exceptional craft. The reading students do should be broad and deep, allowing them to extend their knowledge of particular subjects as well as learn about the features of texts written for different disciplines, audiences, and purposes. While no sampling can do justice to the numerous ways in which different authors craft complex prose, as a collection the exemplar texts below illustrate the level of complexity that college- and career-ready students should be able to handle independently by the end of high school. Texts in translation have not been included in this draft but will be part of future drafts.

How Text Complexity was Determined

In addition to surveys of required reading in twelfth grade and the first year of college as well as consultations with experts, two leading measurement systems were used to help make the selections below. The first system—a methodology described by Jeanne Chall and her coauthors in *The Qualitative Assessment of Text Difficulty*—employs trained raters to measure the sophistication of vocabulary, density of ideas, and syntactic complexity in a text as well as the general and subject-specific knowledge and the level of reasoning required for understanding it. The second system, Coh-Metrix, incorporates into its computer-based analysis more than sixty specific indices of syntax, semantics, readability, and cohesion to assess text complexity. Central to its assessment are measures of text cohesiveness, which is the degree to which the text uses explicit markers to link ideas. By analyzing the degree to which those links are missing in a text—and therefore the degree to which a reader must make inferences to connect ideas—this measure gauges a key factor in the comprehension demand of a text.

The two methods described above have limitations. The complexity of poems (such as "O Captain! My Captain!") cannot be assessed by Coh-Metrix because poetry adheres to different rules of construction than does prose. Similarly, while individual stories in the sample *New York Times* front pages can be measured for complexity by Coh-Metrix, the method does not capture how the electronic environment enhances or detracts from readability. However, for those exemplar texts whose complexity could be measured by both systems, comparable results were yielded by Coh-Metrix and the Chall method.

Note: The samples of complex text are supplemented by brief performance examples that further clarify the meaning of the standards. These illustrate specifically the application of the performance standards to texts of sufficient complexity, quality, and range. Relevant standards are noted in brackets following each sample performance.

Notes on Illustrative Text #1

Pride and Prejudice by Jane Austen

Jane Austen's *Pride and Prejudice* is a sophisticated literary text featuring multiple plotlines, a style and word choice reflective of its time period and setting, and subtle relationships among characters; the excerpt here can only illustrate some of the complexities that readers of the full work will encounter. The novel's opening sentence—"It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife"—signals that today's readers will need to employ literary imagination and historical context to re-create for themselves a world largely in the past. The novel's style is elaborate, with many lengthy and, to the modern ear, formal-sounding sentences typical of the period during which the novel was written. While the dialogue is less formal than much of the surrounding text, words and phrases such as *let* (to mean "rent" or "lease") and *chaise and four* (referring to a type of carriage) mark the novel's setting. The excerpt suggests also the kind of close reading of the subtleties of character that readers must perform. The banter between Mr. and Mrs. Bennet reveals both affection and difference of opinion, and it offers clues to the mores of well-to-do English society in the early nineteenth century.

Sample performance aligned with the Core Standards

Students analyze the first impressions given of Mr. and Mrs. Bennet in the first chapter of *Pride and Prejudice* based on how the characters are described, what they say and do, and how they interact. Students compare these first impressions with their later understanding based on how the characters develop throughout the novel. [R-6]

Illustrative Text #1

from *Pride and Prejudice*

Chapter 1

It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife.

However little known the feelings or views of such a man may be on his first entering a neighbourhood, this truth is so well fixed in the minds of the surrounding families that he is considered as the rightful property of some one or other of their daughters.

"My dear Mr. Bennet," said his lady to him one day, "have you heard that Netherfield Park is let at last?"

Mr. Bennet replied that he had not.

"But it is," returned she; "for Mrs. Long has just been here, and she told me all about it."

Mr. Bennet made no answer.

"Do not you want to know who has taken it?" cried his wife impatiently.

"*You* want to tell me, and I have no objection to hearing it."

This was invitation enough.

"Why, my dear, you must know, Mrs. Long says that Netherfield is taken by a young man of large fortune from the north of England; that he came down on Monday in a chaise and four to see the place, and was so much delighted with it, that he agreed with Mr. Morris immediately; that he is to take possession before Michaelmas, and some of his servants are to be in the house by the end of next week."

"What is his name?"

"Bingley."

"Is he married or single?"

"Oh! single, my dear, to be sure! A single man of large fortune; four or five thousand a year. What a fine thing for our girls!"

"How so? how can it affect them?"

"My dear Mr. Bennet," replied his wife, "how can you be so tiresome! You must know that I am thinking of his marrying one of them."

"Is that his design in settling here?"

"Design! nonsense, how can you talk so! But it is very likely that he *may* fall in love with one of them, and therefore you must visit him as soon as he comes."

"I see no occasion for that. You and the girls may go, or you may send them by themselves, which perhaps will be still better, for as you are as handsome as any of them, Mr. Bingley might like you the best of the party."

"My dear, you flatter me. I certainly *have* had my share of beauty, but I do not pretend to be any thing extraordinary now. When a woman has five grown-up daughters she ought to give over thinking of her own beauty."

"In such cases a woman has not often much beauty to think of."

"But, my dear, you must indeed go and see Mr. Bingley when he comes into the neighbourhood."

"It is more than I engage for, I assure you."

"But consider your daughters. Only think what an establishment it would be for one of them. Sir William and Lady Lucas are determined to go, merely on that account, for in general, you know, they visit no new-comers. Indeed you must go, for it will be impossible for us to visit him if you do not."

"You are over-scrupulous surely. I dare say Mr. Bingley will be very glad to see you; and I will send a few lines by you to assure him of my hearty consent to his marrying whichever he chuses of the girls: though I must throw in a good word for my little Lizzy."

"I desire you will do no such thing. Lizzy is not a bit better than the others; and I am sure she is not half so handsome as Jane, nor half so good-humoured as Lydia. But you are always giving *her* the preference."

"They have none of them much to recommend them," replied he; "they are all silly and ignorant, like other girls; but Lizzy has something more of quickness than her sisters."

"Mr. Bennet, how can you abuse your own children in such a way! You take delight in vexing me. You have no compassion on my poor nerves."

"You mistake me, my dear. I have a high respect for your nerves. They are my old friends. I have heard you mention them with consideration these twenty years at least."

"Ah! you do not know what I suffer."

"But I hope you will get over it, and live to see many young men of four thousand a year come into the neighbourhood."

"It will be no use to us if twenty such should come, since you will not visit them."

"Depend upon it, my dear, that when there are twenty, I will visit them all."

Mr. Bennet was so odd a mixture of quick parts, sarcastic humour, reserve, and caprice, that the experience of three-and-twenty years had been insufficient to make his wife understand his character. *Her* mind was less difficult to develope. She was a woman of mean understanding, little information, and uncertain temper. When she was discontented she fancied herself nervous. The business of her life was to get her daughters married; its solace was visiting and news.

Notes on Illustrative Text #2

"O Captain! My Captain!" by Walt Whitman

Though poetry's complexity cannot be assessed by the measures of readability used for the prose exemplars, "O Captain! My Captain!" by Walt Whitman clearly has many of the features of complex texts listed in the Standards for the Range and Content of Student Reading. Modern readers must work to understand what would have been obvious to readers in 1865: "O Captain! My Captain!" is an extended-metaphor poem intended to convey Whitman's and the North's grief over the assassination of Abraham Lincoln so near the conclusion of hostilities in the Civil War. Every element in the poem stands for something else, with the captain representing Lincoln, the ship representing the Union (or the "ship of state"), the voyage representing the war, and so on. Historical context, along with skill in reading literature, is thus particularly important to interpreting this text.

Sample performance aligned with the Core Standards

Students apply knowledge gained from reading the *New York Times* articles on Lincoln's assassination to their understanding of the poem "O Captain! My Captain!" Specifically, students draw on the description of the crowd's response to the attack on Lincoln to inform their understanding of Whitman's poem. [R-17]

Illustrative Text #2

"O Captain! My Captain!" by Walt Whitman

O Captain! my Captain! our fearful trip is done,
The ship has weather'd every rack, the prize we sought
is won,
The port is near, the bells I hear, the people all exulting,
While follow eyes the steady keel, the vessel grim and daring,
But O heart! heart! heart!
O the bleeding drops of red,
Where on the deck my Captain lies,
Fallen cold and dead.

O Captain! my Captain! rise up and hear the bells;
Rise up—for you the flag is flung—for you the bugle trills,
For you bouquets and ribbon'd wreaths—for you the shores
a-crowding,
For you they call, the swaying mass, their eager faces turning,
Here, Captain! dear father!
This arm beneath your head;
It is some dream that on the deck
You've fallen cold and dead.

My Captain does not answer, his lips are pale and still
My father does not feel my arm, he has no pulse nor will,
The ship is anchor'd safe and sound, its voyage closed
and done,
From fearful trip, the victor ship comes in with object won;
Exult, O shores, and ring O bells!
But I with mournful tread
Walk the deck my Captain lies,
Fallen cold and dead.

Notes on Illustrative Text #3

The front page of the *New York Times*, April 15, 1865

The challenge posed to a modern reader by the front page of the *New York Times* on April 15, 1865, is significant in terms of format, timeliness, and point of view. Unlike the graphically heavy front page of modern newspapers, this 1865 *New York Times* front page is mostly uninterrupted columns of text. The reader is obviously expected to proceed from top to bottom and left to right across the page, but little other guidance is provided. Because the assassination of Lincoln was still "breaking news" as this edition of the *Times* would have gone to press, some details of the event would have not yet been known; readers will have to sort out what they know about the assassination from what the people reading the paper on that Saturday morning would just have been learning. Three accounts of the events rather than one are provided here, and the sourcing and tone vary greatly. Certain details found in one place are contradicted in another: the "Detail of the Occurrence," for example, suggests that Lincoln may not have been mortally wounded, but the main headline in the top left-hand corner of the page states "No Hopes Entertained of His Recovery." While the first two accounts aim at a certain objectivity, the third begins with a flourish that may surprise readers more used to a restrained style of journalism: "A stroke from Heaven laying the whole of the city in instant ruin could not have startled us as did the word that broke from Ford's Theatre a half hour ago that the President had been shot."

Sample performance aligned with the Core Standards

Students analyze how the three different accounts on the front page portray Lincoln's assassination, including which details are similar or different. [R-15]

Illustrative Text #3: The front page of the *New York Times*, April 15, 1865

http://timesmachine.nytimes.com/browser/1865/04/15/P1

The New-York Times.

VOL. XIV.....NO. 4230. NEW-YORK, SATURDAY, APRIL 15, 1865. PRICE FOUR CENTS

AWFUL EVENT.

President Lincoln Shot by an Assassin.

The Deed Done at Ford's Theatre Last Night.

THE ACT OF A DESPERATE REBEL

The President Still Alive at Last Accounts.

No Hopes Entertained of His Recovery.

Attempted Assassination of Secretary Seward.

DETAILS OF THE DREADFUL TRAGEDY.

[OFFICIAL.]

War Department, Washington, April 15-1:30 A.M.

Maj.-Gen. Dix:

This evening at about 9:30 P.M., at Ford's Theatre, the President, while sitting in his private box with Mrs. Lincoln, Mrs. Harris, and Major Rathbum, was shot by an assassin, who suddenly entered the box and approached behind the President.

The assassin then leaped upon the stage, brandishing a large dagger or knife, and made his escape in the rear of the theater.

The entire city is right presents a scene of wild excitement, accompanied by the most expressions of indignation, and the profoundest sorrow—many shed tears. The military authorities have dispatched mounted patrols in every direction, in order, if possible, to arrest the assassins. The whole metropolitan police are likewise vigilant for the same purpose.

The attacks, both at the theatre and at Secretary Seward's house, took place about the same hour—10 o'clock—thus showing a premeditated plan to assassinate those gentlemen. Some evidence of the guilt of the party who attacked the President are in the possession of the police.

Vice-President Johnson is in the city, and is comfortable, but greatly in trouble.

ANOTHER ACCOUNT.

Special Dispatch to the New-York Times.

Washington, Friday, April 14, 11:15 P. M.

A stroke from Heaven laying the whole of the city in instant ruin could not have startled us as did the word that broke from Ford's Theatre a half hour ago that the President had been shot. It flew everywhere in five minutes, and set five thousand people in swift and excited motion on the instant.

DETAIL OF THE OCCURRENCE.

Washington, Friday, April 14- 12:30 A.M.

The President was shot in a theater tonight, and is, perhaps, mortally wounded. Secretary Seward was also assassinated.

SECOND DISPATCH.

Washington, Friday, April 14. President Lincoln and wife, with other friends, this evening visited Ford's Theater for the purpose of witnessing the performance of the "American Cousin."

EUROPEAN NEWS.

TWO DAYS LATER BY THE EUROPA.

The Insult to Our Cruisers by Portugal.

The Mexican Minister at Lisbon Demands Satisfaction.

Dismissal of the Commander of Fort Belem Requested.

Further Advance by Five-Twenties.

FINANCIAL AND COMMERCIAL.

Notes on Illustrative Text #4

The Declaration of Independence

The Declaration of Independence represents the kind of rich primary source material students should be able to read on their own by the end of high school. Though some of the lines ("We hold these truths . . .") are familiar to most American readers, the case against Great Britain that the Declaration lays out, expressed in elevated, sometimes archaic language (*unalienable*, *hath*, *usurpations*), requires careful examination to follow in its particulars. The beginning of the document, excerpted here, poses a reading challenge partly because of its philosophical abstractness. The first three sentences, although formally divided, are one continuous list of propositions ("truths") about the nature of government and the rights of the people. Further complicating the reading is that there is little explicit cohesion between sentences—links supplied by words and phrases such as "for example," "moreover," or "in addition"—to help readers understand the relationship between the ideas being expressed.

Sample performance aligned with the Core Standards

Students compare the argument that the Declaration makes justifying revolution to Martin Luther King, Jr.'s defense of civil disobedience in *Letter from Birmingham Jail*. [R-15]

Illustrative Text #4

from The Declaration of Independence

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. —That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, —That whenever any Form of Government becomes destructive to these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new guards for their future security. —Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

Notes on Illustrative Text #5

Letter from Birmingham Jail by Martin Luther King, Jr.

Martin Luther King, Jr.'s, *Letter from Birmingham Jail* presents many challenges to the reader in terms of its format, purpose, tone, use of allusions, and language. Apart from letters to the editor (most of which are relatively short), public letters such as King's are uncommon today. The purpose of the text may also be confusing: King is ostensibly addressing his "Fellow Clergymen," but skilled readers will reasonably infer that King's message is intended for a broader audience. Though the tone of the text is measured, King's passion for his cause comes through. The author frequently points outside the *Letter* itself through allusions to other texts, including the Hebrew and Christian scriptures. Moreover, King uses sophisticated vocabulary (*cognizant*, *mutuality*, *provincial*, *gainsaying*) and figurative language (*garment of destiny*) throughout his text. However, the piece is both coherent in that its sequence is signaled ("While confined here . . . But more basically . . . Moreover, I am cognizant . . .") and cohesive in that its clauses and sentences are logically linked for the reader ("Just as the prophets . . . and just as the Apostle Paul . . . so am I compelled . . .").

Sample performance aligned with the Core Standards

Students evaluate the reasoning and rhetoric of the three very different arguments King makes to defend his being in Birmingham. Students assess the different kinds of evidence he uses to support each argument. [R-14]

Illustrative Text #5

from *Letter from Birmingham Jail**

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My Dear Fellow Clergymen:

While confined here in the Birmingham city jail, I came across your recent statement calling my present activities "unwise and untimely." Seldom do I pause to answer criticism of my work and ideas. If I sought to answer all the criticisms that cross my desk, my secretaries would have little time for anything other than such correspondence in the course of the day, and I would have no time for constructive work. But since I feel that you are men of genuine good will and that your criticisms are sincerely set forth, I want to try to answer your statements in what I hope will be patient and reasonable terms.

I think I should indicate why I am here In Birmingham, since you have been influenced by the view which argues against "outsiders coming in." I have the honor of serving as president of the Southern Christian Leadership Conference, an organization operating in every southern state, with headquarters in Atlanta, Georgia. We have some eighty-five affiliated organizations across the South, and one of them is the Alabama Christian Movement for Human Rights. Frequently we share staff, educational and financial resources with our affiliates. Several months ago the affiliate here in Birmingham asked us to be on call to engage in a nonviolent direct-action program if such were deemed necessary. We readily consented, and when the hour came we lived up to our promise. So I, along with several members of my staff, am here because I was invited here I am here because I have organizational ties here.

But more basically, I am in Birmingham because injustice is here. Just as the prophets of the eighth century B.C. left their villages and carried their "thus saith the Lord" far beyond the boundaries of their home towns, and just as the Apostle Paul left his village of Tarsus and carried the gospel of Jesus Christ to the far corners of the Greco-Roman world, so am I. compelled to carry the gospel of freedom beyond my own home town. Like Paul, I must constantly respond to the Macedonian call for aid.

Moreover, I am cognizant of the interrelatedness of all communities and states. I cannot sit idly by in Atlanta and not be concerned about what happens in Birmingham. Injustice anywhere is a threat to justice everywhere. We are caught in an inescapable network of mutuality, tied in a single garment of destiny. Whatever affects one directly, affects all indirectly. Never again can we afford to live with the narrow, provincial "outside agitator" idea. Anyone who lives inside the United States can never be considered an outsider anywhere within its bounds.

*As reprinted in *Why We Can't Wait* by King, Jr., M. L. (2000). New York City: Signet Classics.

Notes on Illustrative Text #6

Toni Morrison's Nobel lecture, 1993

Toni Morrison's Nobel lecture, though originally delivered orally, can be read on the page as a complex work of analysis and criticism. Its structure, syntax, imagery, language, and density of ideas contribute to the challenge of studying it in this manner. As this excerpt shows, Morrison begins with a folktale. While the "once upon a time" opening may lead readers into thinking that the lecture will primarily be in narrative form, Morrison uses the tale mainly as a springboard for an abstract, allegorical discussion of language, writing, and those who have no voice in society. Morrison often employs sophisticated sentences that require patience and concentration to follow. Readers may recognize places where Morrison varies sentence patterns to change pace and rhythm—particularly important to the oral delivery of the text. The images Morrison creates are powerful and poetic, the diction is elevated and academic, and the word choice is metaphorical and unconventional: "Official language smitheryed to sanction ignorance and preserve privilege is a suit of armor polished to shocking glitter, a husk from which the knight departed long ago." The richness and abstractness of the ideas in the lecture mean that rereadings may be necessary to comprehend and evaluate the ideas fully.

Sample performance aligned with the Core Standards

Students determine what Morrison means when she compares language to "a bird in the hand," including the different connotations of this phrase that she develops throughout the lecture. Students also explore what Morrison means by saying that both the bird and language can be "dead or alive." [R-7]

Illustrative Text #6

from Toni Morrison's Nobel lecture, 1993

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"Once upon a time there was an old woman. Blind but wise." Or was it an old man? A guru, perhaps. Or a griot soothing restless children. I have heard this story, or one exactly like it, in the lore of several cultures.

"Once upon a time there was an old woman. Blind. Wise."

In the version I know the woman is the daughter of slaves, black, American, and lives alone in a small house outside of town. Her reputation for wisdom is without peer and without question. Among her people she is both the law and its transgression. The honor she is paid and the awe in which she is held reach beyond her neighborhood to places far away; to the city where the intelligence of rural prophets is the source of much amusement.

One day the woman is visited by some young people who seem to be bent on disproving her clairvoyance and showing her up for the fraud they believe she is. Their plan is simple: they enter her house and ask the one question the answer to which rides solely on her difference from them, a difference they regard as a profound disability: her blindness. They stand before her, and one of them says, "Old woman, I hold in my hand a bird. Tell me whether it is living or dead."

She does not answer, and the question is repeated. "Is the bird I am holding living or dead?"

Still she doesn't answer. She is blind and cannot see her visitors, let alone what is in their hands. She does not know their color, gender or homeland. She only knows their motive.

The old woman's silence is so long, the young people have trouble holding their laughter.

Finally she speaks and her voice is soft but stern. "I don't know", she says. "I don't know whether the bird you are holding is dead or alive, but what I do know is that it is in your hands. It is in your hands."

Her answer can be taken to mean: if it is dead, you have either found it that way or you have killed it. If it is alive, you can still kill it. Whether it is to stay alive, it is your decision.

Whatever the case, it is your responsibility.

For parading their power and her helplessness, the young visitors are reprimanded, told they are responsible not only for the act of mockery but also for the small bundle of life sacrificed to achieve its aims. The blind woman shifts attention away from assertions of power to the instrument through which that power is exercised.

Speculation on what (other than its own frail body) that bird-in-the-hand might signify has always been attractive to me, but especially so now thinking, as I have been, about the work I do that has brought me to this company. So I choose to read the bird as language and the woman as a practiced writer. She is worried about how the language she dreams in, given to her at birth, is handled, put into service, even withheld from her for certain nefarious purposes. Being a writer she thinks of language partly as a system, partly as a living thing over which one has control, but mostly as agency—as an act with consequences. So the question the children put to her: "Is it living or dead?" is not unreal because she thinks of language as susceptible to death, erasure; certainly imperiled and salvageable only by an effort of the will. She believes that if the bird in the hands of her visitors is dead the custodians are responsible for the corpse. For her a dead language is not only one no longer spoken or written, it is unyielding language content to admire its own paralysis. Like statist language, censored and censoring. Ruthless in its policing duties, it has no desire or purpose other than maintaining the free range of its own narcotic narcissism, its own exclusivity and dominance. However moribund, it is not without effect for it actively thwarts the intellect, stalls conscience, suppresses human potential. Unreceptive to interrogation, it cannot form or tolerate new ideas, shape other thoughts, tell another story, fill baffling silences. Official language smitheryed to sanction ignorance and preserve privilege is a suit of armor polished to shocking glitter, a husk from which the knight departed long ago. Yet there it is: dumb, predatory, sentimental. Exciting reverence in schoolchildren, providing shelter for despots, summoning false memories of stability, harmony among the public.

Notes on Illustrative Text #7

Inquiry into Life, 12th edition, by Sylvia S. Mader

These excerpts, and the prominent college-level biology textbook from which they are drawn, represent some of the challenges presented by complex writing in natural science, including discipline-specific terms (*covalent bond*, *plasma membrane*, *neurotransmitter*), everyday language used in specialized ways (*shell*, *channel*), abbreviations (H^+, *AChE*), and chains of cause-effect relationships that together describe sometimes elaborate processes. Although the figures the author, Sylvia S. Mader, refers to in the text are not included with these excerpts, students reading the larger work will have to integrate words, illustrations, and diagrams to make full sense of the ideas and concepts she describes. For these reasons and others, comprehension may be difficult for readers who have not had experience independently reading similar kinds of text and who lack a knowledge base in the subject. The author does employ a number of cohesive features to help readers understand the terminology and to link ideas. She repeats content words to let readers follow the flow of ideas; she sets up contrastive situations to illustrate the ideas (within, for example, the first and the third paragraphs below); and she uses transitional links ("In some synapses . . . In other synapses . . .") to help readers construct meaning.

Sample performance aligned with the Core Standards

Students discern the most important information in the description of covalent bonding and provide an accurate summary of the concept. [R-3]

Illustrative Text #7

from *Inquiry into Life*, 12th edition

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A **covalent bond** results when two atoms share electrons in such a way that each atom has an octet of electrons in the outer shell. In a hydrogen atom, the outer shell is complete when it contains two electrons. If hydrogen is in the presence of a strong electron acceptor, it gives up its electron to become a hydrogen ion (H^+). But if this is not possible, hydrogen can share with another atom and thereby have a completed outer shell. For example, one hydrogen atom will share with another hydrogen atom. Their two orbitals overlap, and the electrons are shared between them. Because they share the electron pair, each atom has a completed outer shell.

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The passage of salt (NaCl) across a plasma membrane is of primary importance to most cells. The chloride ion (Cl^-) usually crosses the plasma membrane because it is attracted by positively charged sodium ions (Na^+). First sodium ions are pumped across a membrane, and then chloride ions simply diffuse through channels that allow their passage.

As noted in Figure 4.2*a*, the genetic disorder cystic fibrosis results from a faulty chloride channel. Ordinarily, after chloride ions have passed though the membrane, sodium ions (Na^+) and water follow. In cystic fibrosis, Cl^- transport is reduced, and so is the flow of Na^+ and water.

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Once a neurotransmitter has been released into a synaptic cleft and has initiated a response, it is removed from the cleft. In some synapses, the postsynaptic membrane contains enzymes that rapidly inactivate the neurotransmitter. For example, the enzyme **acetylcholinesterase (AChE)** breaks down acetylcholine. In other synapses, the presynaptic membrane rapidly reabsorbs the neurotransmitter, possibly for repackaging in synaptic vesicles or for molecular breakdown. The short existence of neurotransmitters at a synapse prevents continuous stimulation (or inhibition) of postsynaptic membranes.

Notes on Illustrative Text #8

Sample business memo (ACT WorkKeys Reading for Information Test)

Though not a typical kind of reading in high school classrooms, the business communication, such as the one sampled here, is a form that career-ready students will need to be able to comprehend independently. This text, taken from ACT's WorkKeys Reading for Information Test, is challenging in large part because, like many such communications, it contains important, detailed information intended for a specialized audience. Structurally, the text offers little guidance on how it should be read. Potentially vital details appear throughout and are mingled with other details irrelevant to some readers (e.g., those without children). Even the paragraphing is somewhat inconsistent, especially between the first and second paragraphs. While the sentences are not particularly long and the language is not overly technical, the density of information and its lack of prioritization make this a complex text.

Sample performance aligned with the Core Standards

Students infer from the memo the conditions under which children who are under nineteen are not covered by the health plan. [R-1]

Illustrative Text #8

Sample business memo

WorkKeys Reading for Information Test has been reproduced with permission of ACT, Inc.

DETERMINING ELIGIBILITY FOR MEDICAL COVERAGE

All full-time employees of the company who work an average of at least 30 hours per week are eligible under this plan. Coverage begins on the first day of the month following the 30 days of active full-time employment. If employees enroll within 31 days of the date they are eligible, medical evidence of good health is not required. Temporary and part-time employees are not eligible. Employees are no longer eligible under this plan one month after the date they begin active duty in the armed forces of any country and continuing for the duration of their service.

If employees enroll their dependents within 31 days of the date they become eligible, medical evidence of good health is not required. If they do not, they will be required to submit evidence of good health for each dependent, at their expense, which is satisfactory to the company.

The following dependents are eligible under this plan: employees' spouses, employees' unmarried children under age 19, employees' unmarried dependent children under age 23 who are attending trade school, college, or university on a full-time basis, or employees' unmarried disabled children age 19 and over. Coverage ceases when spouses or children cease to be dependent upon employees for support. In the case of employees' spouses this is if they are legally separated or divorced. In the case of disabled children, this is when they are no longer disabled. Coverage will cease when dependents have served in the armed forces of any country for more than one month, or when maximum benefits have been paid.

Notes on Illustrative Text #9

FedViews, July 9, 2009, by Mary C. Daly
(The Federal Reserve Bank of San Francisco's Web site)

This text illustrates some of the difficulties posed by integrating information gained from words and graphics. This sort of challenge is common in writing designed to inform or explain, including writing in the workplace. The bullet point format used here means that the kind of explicit transitions between ideas typically found in prose are missing; readers will have to infer relationships between the points made by the author, Mary C. Daly, and synthesize the information into a coherent whole. Readers will furthermore have to analyze both the words and the graphics, integrate the information, and check to see whether each source of information supports the other. Daly also uses a great deal of specialized language; the terms *feedback loop*, *credit availability*, and *barriers to credit* all appear in just the first bullet point here.

Sample performance aligned with the Core Standards

Students synthesize information drawn from the text as well as the graphs in order to gain an overarching view of the economy on July 9, 2009. [R-11]

Illustrative Text #9

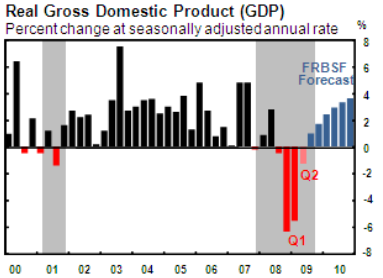
from FedViews, July 9, 2009

Reprinted from the Federal Reserve Bank of San Francisco's FedViews of July 9, 2009. The opinions expressed in this article do not necessarily reflect the views of the management of the Federal Reserve Bank of San Francisco, or of the Board of Governors of the Federal Reserve System.

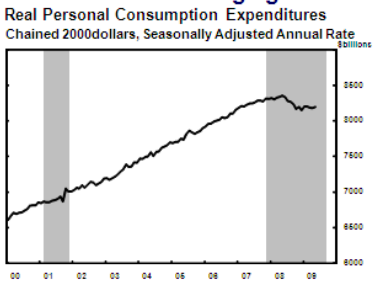
Mary C. Daly, vice president and director of the Center for the Study of Innovation and Productivity at the Federal Reserve Bank of San Francisco, states her views on the current economy and the outlook.

- Financial markets are improving, and the crisis mode that has characterized the past year is subsiding. The adverse feedback loop, in which losses by banks and other lenders lead to tighter credit availability, which then leads to lower spending by households and businesses, has begun to slow. As such, investors' appetite for risk is returning, and some of the barriers to credit that have been constraining businesses and households are diminishing.
- Income from the federal fiscal stimulus, as well as some improvement in confidence, has helped stabilize consumer spending. Since consumer spending accounts for two-thirds of all economic activity, this is a key factor affecting our forecast of growth in the third quarter.
- The gradual nature of the recovery will put additional pressure on state and local budgets. Following a difficult 2009, especially in the West, most states began the 2010 fiscal year on July 1 with even larger budget gaps to solve.
- Still, many remain worried that large fiscal deficits will eventually be inflationary. However, a look at the empirical link between fiscal deficits and inflation in the United States shows no correlation between the two. Indeed, during the 1980s, when the United States was running large deficits, inflation was coming down.

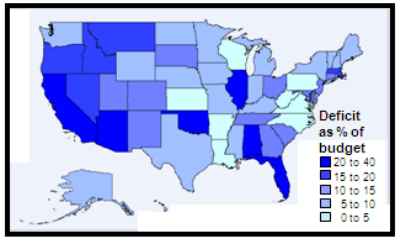
Modest recovery to begin in Q3



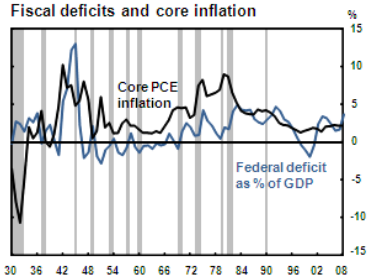
Consumers hanging on



State budget gaps pervasive in 2009



No link between deficits and inflation



Notes on Illustrative Text #10

The front page of the *New York Times*, Web version, August 18, 2009, 9:03 a.m. ET

The challenge offered by this online text and others like it is very different from that offered by a complex continuous text in, say, the sciences. The brief passages are not conceptually difficult, the language is not technical or esoteric, and the sentences are not particularly complex. But these characteristics belie the complexity of the reading task. An online text of this kind requires readers to apply their print-reading skills in tandem with their knowledge of how to use online periodicals. The editors and designers have assigned levels of importance to individual stories and images, as measured by their size and position in the layout. The page itself uses words, numbers, icons, and other visual elements (e.g., line, color, and shape) to guide readers further. Headings in various colors direct readers to particular sections (OPINION, MARKETS, HEALTH), while links direct readers to particular stories ("Taliban Talks Are Key Issue in Afghan Vote"). Time markers ("3 minutes ago") help readers assess how new the information in a given story is. The text requires readers to make choices about which links to follow based on their understanding of how online text is typically structured and on a minimum of additional information (e.g., an icon of a camera, a drop-down menu in an ad).

Sample performance aligned with the Core Standards

Students select an article and use search terms and other features of the online text to research a specific aspect of the subject in more depth. [R-12]

Illustrative Text #10

The front page of the *New York Times*, Web version, August 18, 2009, 9:03 a.m. ET

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Sample of Works Consulted

I. College Readiness

A. Achieve, Inc. (2004). The American Diploma Project, *Ready or Not: Creating a High School Diploma that Counts*. Washington, DC: Achieve, Inc. (PDF)

B. Achieve, Inc. (2008) *Out of Many, One: Towards Rigorous Common Core Standards from the Ground Up*. Washington, DC: Achieve, Inc. (PDF)

C. ACT. (2009). *ACT College Ready English Standards*. Iowa City, IA: ACT. (PDF)

D. ACT. (2006). *ACT National Curriculum Survey 2005-2006*. Iowa City, IA: ACT. (PDF)

E. ACT. (2007). *Aligning Postsecondary Expectations and High School Practice: The Gap Defined (Policy Implications of the ACT National Curriculum Survey Results 2005-2006)*. Iowa City, IA: ACT. (PDF)

F. ACT. (2006). *Reading Between the Lines: What the ACT Reveals about College Readiness in Reading.* Iowa City, IA: ACT. (PDF)

G. ACT. (2006). *Ready for College and Ready for Work: Same or Different?* Iowa City, IA: ACT. (PDF)

H. ACT. (2007). *Rigor at Risk: Reaffirming Quality in the High School Core Curriculum.* Iowa City, IA: ACT. (PDF)

I. College Board (2008) *AP English Language and Composition and English Literature and Composition Course Description.* New York, NY: College Board. (PDF)

J. College Board (2009) *AP European History Course Description*, New York, NY: College Board. (PDF)

K. College Board (2009) *AP World History Course Description*, New York, NY: College Board. (PDF)

L. College Board. (2006). *College Board Standards for College Success.* New York, NY: College Board. (PDF)

M. Conley, D.T. (2003) *Understanding University Success: A Report from Standards for Success.* Eugene, OR: Center for Educational Policy Research.

N. Intersegmental Committee of the Academic Senates (ICAS). (2002) *Academic Literacy: A Statement of Competencies Expected of Students Entering California's Public Colleges and Universities.* (PDF)

O. Marzano, R.J. and Kendall, J.S. (2007). *The New Taxonomy of Educational Objectives.* Second Edition. Thousand Oaks, CA: Corwin Press.

P. Morreale, S. & Pearson, J. (2008) "Why communication education is important: The centrality of the discipline in the 21st century." *Communication Education*, 57(2), 224-240.

Q. Milewski, G.B., Johnsen, D., Glazer, N., & Kubota, M. (2005). *A Survey to Evaluate the Alignment of the New SAT® Writing and Critical Reading Sections to Curricula and Instructional Practices.* New York, NY: College Entrance Examination Board. (PDF)

R. *Virginia Postsecondary Outreach Campaign and Data Collection, Essential English Skills Analysis*. (Achieve).

II. Career Readiness

A. Achieve, Inc. (2004). *The American Diploma Project, Ready or Not: Creating a High School Diploma that Counts*. Washington, DC: Achieve, Inc. (PDF)

B. ACT. (2007). *Aligning Postsecondary Expectations and High School Practice: The Gap Defined (Policy Implications of the ACT National Curriculum Survey Results 2005-2006)*. Iowa City, IA: ACT. (PDF)

C. ACT. (2006). *Ready for College and Ready for Work: Same or Different?* Iowa City, IA: ACT. (PDF)

D. ACT. (2007). *Rigor at Risk: Reaffirming Quality in the High School Core Curriculum*. Iowa City, IA: ACT. (PDF)

E. Casner-Lotto, J., Rosenblum, E., and Wright, M., (2009). *The Ill-Prepared Workforce: Exploring the Challenges of Employer-Provided Workforce Readiness Training.* The Conference Board.

F. Florida American Diploma Project Survey Results. (Achieve).

G. Hawai'i Career Ready Study. (2007). Commissioned by the Hawai'i P-20 Initiative.(PDF)

H. *Missouri Career Prep Certificate Program Planning Guide,* Missouri Department of Elementary and Secondary Education. (PDF)

I. Morreale, S. & Pearson, J. (2008) "Why communication education is important: The centrality of the discipline in the 21st century." *Communication Education*, 57(2), 224-240.

J. National Alliance of Business. (2002).*The American Diploma Project Workplace Study*. Washington, DC.

K. Partnership for 21st Century Skills. (2009). *Framework for 21st Century Learning*. Tucson, AZ.

L. Qualifications and Curriculum Authority. (2007). *Functional Skills and Standards.* (PDF)

M. The Conference Board. (2008). *New Graduates' Workforce*

Readiness.

III. International Documents

A. **Alberta, Canada:** *English Language Arts Curriculum Outcomes*, 2003. (PDF)

B. **British Columbia, Canada:** *English Language Arts Integrated Resource Package, Prescribed Learning Outcomes,* 2007. (PDF)

C. **British Columbia:** English Literature 12, Integrated Resource Package, 2003. (PDF)

D. **England:** *English Programme of Study for Key stage 4*, 2007. (PDF)

E. **Finland:** *National Core Curriculum for Upper Secondary Schools* for Mother Tongue and Literature, Finnish as the mother tongue, 2003. (PDF)

F. **Hong Kong:** *English Language Curriculum and Assessment Guide*, 2007. (PDF)

G. **Hong Kong:** Hong Kong Examinations and Assessment Authority, 2007 HKCEE English Language, Recommended Texts for the School-Based Assessment Component. (PDF)

H. **Ireland:** *Leaving Certificate/English Syllabus for Higher Level and Ordinary Level.* (PDF)

I. **Ireland:** Prescribed Material for English in the Leaving Certificate Examination in 2009. (PDF)

J. **New South Wales:** *English Stage 6 Syllabus, 1999.* (PDF)

K. **New South Wales**: 2009-2012 HSC, Prescribed Area of Study, Electives and Texts. (PDF)

L. **Ontario, Canada:** *The Ontario Curriculum, English,* 2007. (PDF)

M. **Singapore:** *English Language Syllabus 2001*. (PDF)

N. **Singapore:** O-Level Literature in English (Syllabus 2015), Prescribed Texts for 2009. (PDF)

O. **Victoria, Australia:** *Victorian Certificate of Education Study Design: English/English as a Second Language*, 2006. (PDF)

P. **Victoria, Australia:** VCE English/ESL Text List 2008-2009, VCE Literature Text List 2008. (PDF)

Q. Achieve, Inc., National Governors Association, Council of Chief State School Officers. (2008). *Benchmarking for Success: Ensuring U.S. Students receive a World Class Education*. Washington, DC. (PDF)

R. Education Commission of the States. (2008). *From Competing to Leading: An International Benchmarking Blueprint*. Denver, CO. (PDF)

S. Education Commission of the States. (2009). *International Benchmarking Toolkit*. Denver, CO. (PDF)

T. Organization for Economic Cooperation and Development. (2003). *The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills*. Paris, France. (PDF)

U. Organization for Economic Cooperation and Development. (2006). *PISA 2006: Science Competencies for Tomorrow's World, Vol. 1.* [see Chapter 6 for reading assessment results] Paris, France. (PDF)

IV. State and Other Standards Documents

A. **California:** *English-Language Arts Content Standards for California Public Schools.* California Department of Education (1997). (PDF)

B. **Florida:** *2006 Sunshine State Standards K-12 Reading and Language Arts.* Florida Department of Education (2006). (PDF)

C. **Georgia:** Georgia ELA Standards (K-12*)*.

D. **Indiana**: Reading List, Grades 9-12. (PDF)

E. **Massachusetts**: *English Language Arts Curriculum Framework.* Massachusetts Department of Education (June 2001). (PDF)

F. **Minnesota:** *Minnesota Academic Standards, Language Arts (K-12),* (2003). (PDF)

G. **Texas:** *Texas College Readiness Standards*, (2008). (PDF)

H. National Council of Teachers of English and the International Reading Association. (1996). *Standards for the English Language Arts.* (PDF)

I. U.S. Dept. of Education. (2009). *Reading Framework for the National Assessment of Educational Progress 2009*. Washington, DC: National Assessment Governing Board. (PDF)

J. U.S. Dept. of Education. (2007). *Writing Framework for the 2011 National Assessment of Educational Progress*. Washington, DC: National Assessment Governing Board. (MS Word Document)

V. Disciplinary Literacy Research

A. Carnegie Council on Advancing Adolescent Literacy. (2010). *Time to act: An agenda for advancing adolescent literacy for college and career success*. New York, NY: Carnegie Corporation of New York. (PDF)

B. Fang, Z., & Schleppegrell, M. J. (2008). *Reading in secondary content areas.* Ann Arbor: University of Michigan Press.

C. Lee, C.D., Spratley, A. (2010). *Reading in the disciplines: The challenges of adolescent literacy*. New York, NY: Carnegie Corporation of New York. (PDF)

D. Moje, E. B., Young, J. P., Readence, J. E., & Moore, D. W. (2000). Reinventing adolescent literacy for new times: Perennial and millennial issues. *Journal of Adolescent and Adult Literacy, 43*, 400–410.

E. Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective.* Mahwah, NJ: Lawrence Erlbaum Associates.

F. Shanahan, T., & Shanahan, C. (2008). Teaching disciplinary literacy to adolescents: Rethinking content-area literacy. *Harvard Educational Review, 78,* 40–59.

College and Career Readiness Standards for Mathematics

Draft for Review and Comment

September 21, 2009

Contents 1

Introduction 2-4

How Evidence Informed Decisions in Drafting the Standards 5, 6

College and Career Readiness Standards for Mathematics 7-19

- Mathematical Practice 8
- Number 9
- Quantity 10
- Expressions 11
- Equations 12
- Functions 13
- Modeling 14
- Shape 15
- Coordinates 16
- Probability 17
- Statistics 18
- Example Tasks online at www.corestandards.org

Sample of Works Consulted 19-22

Introduction

The *College and Career Readiness Standards for Mathematics* consist of three interconnected parts: a Standard for Mathematical Practice, ten Standards for Mathematical Content, and a set of Example Tasks.

The Standard for Mathematical Practice has six Core Practices that describe the way proficient students approach mathematics. Proficient students attend to precision, construct viable arguments, make sense of complex problems and persevere in solving them, look for hidden structure, note regularity in repeated reasoning, and use technology intelligently. This approach to mathematics is an essential part of being ready for college and career.

The Standards for Mathematical Content form the backbone of this document. Each of these ten standards consists of Core Concepts, Core Skills, and a description of the student's Coherent Understanding. Students who encounter the subject with a focus on coherence will be better able to learn more mathematics at a deeper level and be better able to access and apply the mathematics they know. The ten Standards for Mathematical Content pull together topics previously studied and look ahead toward topics in further coursework and training programs.

The Standards for Mathematical Content are designed to draw greater attention to powerful organizing principles in mathematics, such as functional relationships or the laws of arithmetic. They also allow important distinctions to be made more clearly, such as that between Expressions and Equations. And they surface the deep connections that often underlie mathematical coherence, such as the blending of algebra with geometry represented by Coordinates. These ten are not categories or buckets of topics to cover; they are standards. They describe the coherence students need and deserve as they go forward to their mathematical futures.

The third component of the *College and Career Readiness Standards for Mathematics* is a Web-based collection of Example Tasks that exemplifies the variety of performances required. High standards demand that students *use* their knowledge, skills and good practices to solve problems from a variety of contexts, both within mathematics and from the world outside. Example Tasks exemplify the range and variety of use that is expected. Teachers and designers of curriculum and assessment will find in the collection of examples a guide to what these standards mean. Over time, the collection of tasks will grow.

Together, these three components establish an evidence-based standard for college and career readiness. The *College and Career Readiness Standards for Mathematics* have been created with attention to the expectations of the highest achieving countries. They have focus and depth, emphasizing the understanding of and connections among topics that are most important for success regardless of a student's pathway after reaching these standards.

A primary goal of developing these standards is to enable students to achieve *mathematical proficiency* (see sidebar). Students are expected to understand the knowledge described in the Core Concepts and in the Coherent Understandings at a depth that enables them to reason with that knowledge—to analyze, interpret and evaluate mathematical problems, make deductions, and justify results. The Core Skills are meant to be used strategically and adaptively to solve problems. Students' knowledge and skills come to life and take their value when melded with the ways they approach mathematics—as described by the Core Practices.

The specific verbs used to describe concepts and skills in these standards are not meant to limit or indicate levels of any taxonomy. Although using verbs to indicate levels of depth has been a common practice in this country's standards writing, high performing nations do not use verbs in this way. They describe depth and practices first in separate sections of their syllabi. We have adopted the high performing countries' practice of focusing on a clear statement of what mathematics should be learned when writing standards for knowledge and skills.

Instruction, curriculum and assessment designed to achieve these standards should range over all strands of proficiency in *Adding It Up*, all depths of knowledge in Norman L. Webb's Depth of Knowledge taxonomy, all levels of Bloom's Taxonomy, and all levels of cognitive demand. In the Core Skills and Core Practices we have sometimes used terms like "explore" to indicate a lighter treatment with a goal of awareness and experience rather than proficiency. We have used Example Tasks to show the depth of knowledge and deployment of skills expected.

From *Adding it up: Helping children learn mathematics* (National Research Council, 2001, p. 116):

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully. Mathematical proficiency, as we see it, has five components, or strands:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These standards are measurable; that is, they are observable and verifiable through the broad spectrum of student performances that may be assessed during classroom observation, school-based examinations and large-scale testing. The *College and Career Readiness Standards for Mathematics* can guide the development of assessment frameworks that distribute the assessment responsibilities across multiple levels of the educational system: state, district, school and teacher.

Students reaching these levels will be prepared for non-remedial college mathematics courses and will be prepared for training programs for career-level jobs; however, the *College and Career Readiness Standards for Mathematics* should not be construed as grade twelve exit standards. Students interested in STEM fields, and those who wish to go beyond for other reasons, will need to reach these standards before their senior year in order to have time to include additional mathematics. A number of pathways for advanced learning are possible and may be integrated throughout the high school experience and beyond.

The Common Core State Standards Initiative

The *College and Career Readiness Standards for Mathematics* will anchor the next phase of the Common Core State Standards Initiative: development of K–12 Mathematics Standards. Those K–12 Standards are in turn expected to guide the development of a next generation of assessments, developed collaboratively by multiple states. The K–12 Mathematics Standards will serve as a guide and tool for aligning instruction, curriculum, assessment, teacher supports, and systems of accountability. To ensure alignment, the Standard for Mathematical Practice, the Standards for Mathematical Content, and the Example Tasks should all be taken into account.

Overview of the Mathematical Practice Standard

Attend to precision.
Construct viable arguments.
Make sense of complex problems and persevere in solving them.
Look for structure.
Look for and express regularity in repeated reasoning.
Make strategic decisions about the use of technological tools.

Overview of the Mathematical Content Standards

Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra.

Quantity. A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

Expressions. Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.

Equations. An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true.

Functions. Functions model situations where one quantity determines another. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

Shape. From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships.

Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Probability. Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1.

Statistics. Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data.

How Evidence Informed Decisions in Drafting the Standards

The Common Core State Standards Initiative builds on a generation of standards efforts led by states and national organizations. On behalf of the states, we have taken a step toward the next generation of standards that are aligned to college- and career-ready expectations and are internationally benchmarked. These standards are grounded in evidence from many sources that shows that the next generation of standards in mathematics must be focused on deeper, more thorough understanding of more fundamental mathematical ideas and higher mastery of these fewer, more useful skills.

The evidence that supports this new direction comes from a variety of sources. International comparisons show that high performing countries focus on fewer topics and that the U.S. curriculum is "a mile wide and an inch deep." Surveys of college faculty show the need to shift away from high school courses that merely survey advanced topics, toward courses that concentrate on developing an understanding and mastery of ideas and skills that are at the core of advanced mathematics. Reviews of data on student performance show the large majority of U.S. students are not mastering the mile wide list of topics that teachers cover.

The evidence tells us that in high performing countries like Singapore, the gap between what is taught and what is learned is relatively smaller than in Malaysia or the U.S. states. Malaysia's standards are higher than Singapore's, but their performance is much lower. One could interpret the narrower gap in Singapore as evidence that they actually use their standards to manage instruction; that is, Singapore's standards were set within the reach of hard work for their system and their population. Singapore's Ministry of Education flags its webpage with the motto, "Teach Less, Learn More." We accepted the challenge of writing standards that could work that way for U.S. teachers and students: By providing focus and coherence, we could enable more learning to take place at all levels.

However, a set of standards cannot be simplistically "derived" from any body of evidence. It is more accurate to say that we used evidence to inform our decisions. A few examples will illustrate how this was done.

For example, systems of linear equations are covered by all states, yet students perform surprisingly poorly on this topic when assessed by ACT. We determined that systems of linear equations have high coherence value, mathematically; that this topic is included by all high performing nations; and that it has moderately high value to college faculty. Result: We included it in our standards.

A different and more complex pattern of evidence appeared with families of functions. Again we found that students performed poorly on problems related to many advanced functions (trigonometric, logarithmic, quadratic, exponential, and so on). Again we found that a number of states cover them, even though college faculty rated them lower in value. High performing countries include this material, but with different degrees of demand. We decided that we had to carve a careful line through these topics so that limited teaching resources could focus where most important. We decided that students should

develop deep understanding and mastery of linear and exponential functions. They should also have familiarity with other families of functions, and apply their algebraic, modeling and problem solving skills to them—but not develop in-depth technical mastery and understanding. Thus we defined two distinct levels of attention and identified which families of functions got which level of attention.

Why were exponential functions selected for intensive focus in the Functions standard instead of, say, quadratic functions? What tipped the balance was the high coherence value of exponential functions in supporting modeling and their wide utility in work and life. Quadratic functions were also judged to be well supported by expectations defined under Expressions and Equations.

These examples indicate the kind of reasoning, informed by evidence, that it takes to design standards aligned to the demands of college and career readiness in a global economy. We considered inclusion in international standards, requirements of college and the workplace, surveys of college faculty and the business community, and other sources of evidence. As we navigated these sometimes conflicting signals, we always remained aware of the finiteness of instructional resources and the need for deep mathematical coherence in the standards.

At the end of this document, there is a listing of a number of sources that played a role in the deliberations described above and more generally throughout the process to inform our decisions. A hyperlinked version of the bibliography can be found online at www.corestandards.org.

College and Career Readiness Standards for Mathematics

Mathematical Practice

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement.[a] These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures.[b] Taken together with the Standards for Mathematical Content, they support productive entry into college courses or career pathways.

Core Practices · Students can and do:

1 Attend to precision.

Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies noting the assumptions made. They clarify definitions. They state the meaning of the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. Rather than saying, "let v be speed and let t be time," they would say "let v be the speed in meters per second and let t be the elapsed time in seconds from a given starting time." They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits indicate unwarranted precision.

2 Construct viable arguments.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

3 Make sense of complex problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

4 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern. For example, in $x^2 + 5x + 6$ they can see the 5 as $2 + 3$ and the 6 as 2×3. They recognize the significance of an existing line in a geometric figure and can add an auxiliary line to make the solution of a problem clear. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects. For example, by seeing $5 - 3(x - y)^2$ as 5 minus a positive number times a square, they see that it cannot be more than 5 for any real numbers x and y.[b]

5 Look for and express regularity in repeated reasoning.

Mathematically proficient students pay attention to repeated calculations as they carry them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, they might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel in the expansions of $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ leads to the general formula for the sum of a geometric series. As they work through the solution to a problem, proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.[b]

6 Make strategic decisions about the use of technological tools.

Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, ruler, protractor, graphing calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding and estimation strategically, attending to levels of precision, to ensure appropriate levels of approximation and to detect possible errors. They are able to use these tools to explore and deepen their understanding of concepts.

(a) For the importance of students' beliefs about effort, see the National Mathematics Advisory Panel's Report of the Task Group on Learning Processes, p. 4-10 (2008). (b) Cuoco, A., Goldenberg, E. P., and Mark, J., *Journal of Mathematical Behavior*, 15 (4), 375-402, 1996; *Focus in High School Mathematics*. Reston, VA: NCTM, in press; Harel, G., What is Mathematics? A Pedagogical Answer to a Philosophical Question. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics From the Perspective of Mathematicians*, Mathematical Association of America, 2008.

Number

Core Concepts · Students understand that:

A The real numbers include the rational numbers and are in one-to-one correspondence with the points on the number line.

B Quantities can be compared using division, yielding rates and ratios.

C A fraction can represent the result of dividing the numerator by the denominator; equivalent fractions have the same value.

D Place value and the rules of arithmetic form the foundation for efficient algorithms.

A Coherent Understanding of Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra:

- Numbers can be added in any order with any grouping and multiplied in any order with any grouping.
- Adding 0 and multiplying by 1 both leave a number unchanged.
- All numbers have additive inverses, and all numbers except zero have multiplicative inverses.
- Multiplication distributes over addition.

Subtraction and division are defined in terms of addition and multiplication, so are also governed by these rules.

The place value system bundles units into 10s, then 10s into 100s, and so on, providing an efficient way to name large numbers. Subdividing in a similar way extends this to the decimal system, which provides an address system for locating all real numbers on the number line with arbitrarily high accuracy. Place value is the basis for efficient algorithms, reducing much computation to single-digit arithmetic. Mental computation strategies also make opportunistic use of the rules of arithmetic, as when the product $5 \times 177 \times 2$ is computed at a glance to obtain 1770, rather than methodically working from left to right.

An estimate may be more appropriate than an exact value, for example, when you want to know the number of calories in a meal. Often a result is reported using fewer digits than were calculated. A mature number sense includes having rules of thumb about how much accuracy is appropriate and understanding that accuracy to more than a few decimal places often takes substantial effort. Estimation and approximation are also useful in checking calculations.

Rational numbers represented as fractions can be located on the number line by seeing them as numbers expressed in different units; for example, 3/5 is 3 units, where each unit is 1/5. However, rational numbers do not fill out the number line. There are also irrational numbers, such as π or $\sqrt{2}$. Each point on the number line then corresponds to a real number that is either rational or irrational.

Connections to Expressions, Functions and Coordinates. The rules of arithmetic govern the manipulations of expressions and functions. Two perpendicular number lines define the coordinate plane.

Core Skills · Students can and do:

1 Compare numbers and make sense of their magnitude.

Include positive and negative numbers expressed as fractions, decimals, powers, and roots. Limit to square and cube roots. Include very large and very small numbers and the use of scientific notation.

2 Know when and how to use standard algorithms, and perform them flexibly, accurately and efficiently.*

3 Use mental strategies and technology to formulate, represent and solve problems.**

4 Solve multi-step problems involving fractions and percentages.

Include situations such as simple interest, tax, markups/markdowns, gratuities and commissions, fees, percent increase or decrease, percent error, expressing rent as a percentage of take-home pay, and so on.

5 Use estimation and approximation to solve problems.

Include evaluating answers for their reasonableness, detecting errors, and giving answers to an appropriate level of precision.

* This aligns with the concept of procedural fluency as in the National Research Council report *Adding it up: Helping children learn mathematics*. Specifically, "*Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121).

** This aligns with the concept of strategic competence as described in *Adding it up*. "*Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them" (p. 124).

Quantity

Core Concepts · Students understand that:

A The value of a quantity is not specified unless the units are named or understood from the context.

B Quantities can be added and subtracted only when they are of the same type (length, area, speed, etc.).

C Quantities can be multiplied or divided to create new types of quantities, called derived quantities.

A Coherent Understanding of Quantity. A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

The length of a football field and the speed of light are both quantities. If we choose units of miles per second, then the speed of light has a value of approximately 186,000 miles per second. But the speed of light need not be expressed in miles per second; it may be approximated by 3×10^8 meters per second or in any other unit of speed. Bare numerical values such as 186,000 do not describe quantities unless they are paired with units.

Speed (distance divided by time), rectangular area (length multiplied by length), density (mass divided by volume), and population density (number of people divided by land area) are examples of derived quantities, obtained by multiplying or dividing quantities.

It can make sense to add two quantities, such as when a child 51 inches tall grows 3 inches to become 54 inches tall. To be added or subtracted, quantities must be of the same type (length, area, speed, etc.); to add or subtract their values, the quantities must be expressed in the same units. Converting quantities to have the same units is like converting fractions to have a common denominator before adding or subtracting. But, even when quantities have the same units it does not always make sense to add them. For example, if a wooded park with 300 trees per acre is next to a field with 30 trees per acre, they do not have 330 trees per acre.

Doing algebra with units in a calculation reveals the units of the answer, and can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed.

Connections to Number, Expressions, Equations, Functions, Modeling and Statistics. Operations described under Number and Expressions govern the operations one performs on quantities, including the units involved. Quantity is an integral part of any application of mathematics, and has connections to solving problems using data, equations, functions and modeling.

Core Skills · Students can and do:

1 Know when and how to convert units in computations.

Include the addition and subtraction of quantities of the same type expressed in different units; averaging data given in mixed units; converting units for derived quantities such as density and speed.

2 Use and interpret quantities and units correctly in algebraic formulas.

Include specifying units when defining variables and attending to units when writing expressions and equations.

3 Use and interpret quantities and units correctly in graphs and data displays.

Include function graphs, data tables, scatterplots and other visual displays of dimensioned data.

4 Use units as a way to understand problems and to guide the solution of multi-step problems.

Include examples such as acceleration; currency conversions; people-hours; social science measures, such as deaths per 100,000; and general rates, such as points per game.

Expressions

Core Concepts · Students understand that:

A Expressions are constructions built up from numbers, variables, and operations, which have a numerical value when each variable is replaced with a number.

B Complex expressions are made up of simpler expressions.

C The rules of arithmetic can be applied to transform an expression without changing its value.

D Rewriting expressions in equivalent forms serves a purpose in solving problems.

A Coherent Understanding of Expressions. Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.

Reading an expression with comprehension involves analysis of its underlying structure, which may suggest a different but equivalent way of writing it that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. But rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying by a constant factor.

Algebraic manipulations are based on the conventions of algebraic notation and the rules of arithmetic. Heuristic mnemonic devices are not a substitute for procedural fluency. For example, factoring, expanding, collecting like terms, the rules for interpreting minus signs next to parenthetical sums, and adding fractions with a common denominator are all instances of the distributive law; the definitions for negative and rational exponents are based on the extension of the exponent laws for positive integers. The laws of exponents connect multiplication of numbers to addition of exponents and thus express the deep relationship between addition and multiplication captured by the parallel nature of the rules of arithmetic for these operations.

Complex expressions are made up of simpler expressions using arithmetic operations and substitution. When simple expressions within more complex expressions are treated as single quantities, or chunks, the underlying structure of the larger expression may be more evident.

Connections to Equations and Functions. Setting expressions equal to each other leads to equations. Expressions can define functions of the variables that appear in them, with equivalent expressions defining the same function.

Core Skills · Students can and do:

1 See structure in expressions.

For example, recognize: that the expressions $x^4 - y^4$ and $(x+y)^2 - (x-y)^2$ are differences of squares; that there are different ways to rewrite the latter expression, e.g., by expanding and collecting like terms or by factoring as a difference of squares; that p is a common factor in $p + 0.025p$; that an expression in the form $(x-3)^2 + 14$ reveals its minimum value.

2 Manipulate simple expressions.

Show procedural fluency in the following cases: factoring out common terms; factoring expressions with quadratic structure; writing in standard form sums, differences, and products of polynomials. Include completing the square and rewriting in standard form sums, differences, products, and quotients of simple rational expressions; rewriting expressions with negative exponents and those involving square or cube roots of a single term involving exponents.

3 Define variables and write an expression to represent a quantity in a problem.

Include contextual problems.

4 Interpret an expression that represents a quantity in terms of the context.

Include interpreting parts of an expression, such as terms, factors and coefficients.

Equations

Core Concepts · Students understand that:

A An equation is a statement that two expressions are equal.

B The solutions of an equation are the values of the variables that make the resulting numerical statement true.

C The steps in solving an equation are guided by understanding and justified by logical reasoning.

D Equations not solvable in one number system may have solutions in a larger number system.

A Coherent Understanding of Equations. An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true. If the equation is true for all values of the variables, then we call it an identity; identities are often discovered by manipulating one expression into another.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs, which can be graphed in the plane. Equations can be combined into systems to be solved simultaneously.

An equation can be solved by successively transforming it into one or more simpler equations. The process is governed by deductions based on the properties of equality. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, stimulating the formation of expanded number systems (integers, rational numbers, real numbers and complex numbers).

A formula is a type of equation. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \left(\frac{b_1+b_2}{2}\right)h$, can be solved for h using the same deductive process.

Inequalities can be solved in much the same way as equations. Many, but not all, of the properties of equality extend to the solution of inequalities.

Connections to Functions, Coordinates, and Modeling. Equations in two variables may define functions. Asking when two functions have the same value leads to an equation; graphing the two functions allows for the approximate solution of the equation. Equations of lines involve coordinates, and converting verbal descriptions to equations is an essential skill in modeling.

Core Skills · Students can and do:

1 Understand a problem and formulate an equation to solve it.

Extend to inequalities and systems.

2 Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality.

Solve linear equations with procedural fluency. For quadratic equations, include solution by inspection, by factoring, or by using the quadratic formula. Understand that the quadratic formula comes from completing the square. Include simple absolute value equations solvable by direct inspection and by understanding the interpretation of absolute value as distance.

3 Rearrange formulas to isolate a quantity of interest.

Exclude cases that require extraction of roots or inverse functions.

4 Solve systems of equations.

Focus on pairs of simultaneous linear equations in two variables. Include algebraic techniques, graphical techniques and solving by inspection.

5 Solve linear inequalities in one variable and graph the solution set on a number line.

Emphasize solving the associated equality and determining on which side of the solution of the associated equation the solutions to the inequality lie.

6 Graph the solution set of a linear inequality in two variables on the coordinate plane.

Emphasize graphing the associated equation, using a dashed or solid line as appropriate and shading to indicate the half-plane on which the solutions to the inequality lie.

Functions

Core Concepts · Students understand that:

A A function is a rule, often defined by an expression, that assigns a unique output for every input.

B The graph of a function f is a set of ordered pairs $(x, f(x))$ in the coordinate plane.

C Functions model situations where one quantity determines another.

D Common functions occur in families where each member describes a similar type of dependence.

A Coherent Understanding of Functions. Functions model situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because nature and society are full of dependencies between quantities, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a plane to fly 1000 miles is a function of the plane's average ground speed in miles per hour, v; the rule $T(v) = 1000/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of possible inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. The graph of a function is a useful way of visualizing the relationship the function models, and manipulating the expression for a function can throw light on the function's properties.

Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with an initial value of zero describe proportional relationships.

Connections to Expressions, Equations, Modeling and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. The graph of a function f is the same as the solution set of the equation $y = f(x)$. Questions about when two functions have the same value lead to equations, whose solutions can be visualized from the intersection of the graphs. Since functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be modeled effectively using a spreadsheet or other technology.

Core Skills · Students can and do:

1 Recognize proportional relationships and solve problems involving rates and ratios.

Include being able to express proportional relationships as functions.

2 Describe the qualitative behavior of common types of functions using graphs and tables.

Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Use technology to explore the effects of parameter changes on the graphs of linear, power, quadratic, polynomial, simple rational, exponential, logarithmic, sine and cosine, absolute value and step functions.

3 Analyze functions using symbolic manipulation.

Include slope-intercept and point-slope form of linear functions; vertex form of quadratic functions to identify symmetry and find maximums and minimums; factored form to find zeros. Use manipulations as described under Expressions.

4 Use the families of linear and exponential functions to solve problems.

For linear functions $f(x) = mx + b$, understand b as the intercept or initial value and m as the slope or rate of change. For exponential functions $f(x) = a \cdot b^x$, understand a as the intercept or initial value and b as the growth factor.

5 Find and interpret rates of change.

Compute the rate of change of linear functions and make qualitative observations about how the rate of change varies for nonlinear functions.

Modeling

Core Concepts · Students understand that:

A Mathematical models involve choices and assumptions that abstract key features from situations to help us solve problems.

B Even very simple models can be useful.

A Coherent Understanding of Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

A model can be very simple, such as a geometric shape to describe a physical object like a coin. Even so simple a model involves making choices. It is up to us whether to model the solid nature of the coin with a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. For some purposes, we might even choose to adjust the right circular cylinder to model more closely the way the coin deviates from the cylinder.

In any given situation, the model we devise depends on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models we can create and analyze is constrained as well by the limitations of our mathematical and technical skills. For example, modeling a physical object, a delivery route, a production schedule, or a comparison of loan amortizations each requires different sets of tools. Networks, spreadsheets and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical structure might model seemingly different situations.

The basic modeling cycle is one of (1) identifying the key features of a situation, (2) creating geometric, algebraic or statistical objects that describe key features of the situation, (3) analyzing and performing operations on these objects to draw conclusions and (4) interpreting the results of the mathematics in terms of the original situation. Choices and assumptions are present throughout this cycle.

Connections to Quantity, Equations, Functions, Shape, Coordinates and Statistics. Modeling makes use of shape, data, graphs, equations and functions to represent real-world quantities and situations.

Core Skills · Students can and do:

1 Model numerical situations.

Include readily applying the four basic operations in combination to solve multi-step quantitative problems with dimensioned quantities; making estimates to introduce numbers into a situation and get problems started; recognizing proportional or near-proportional relationships and analyzing them using characteristic rates and ratios.

2 Model physical objects with geometric shapes.

Include common objects that can reasonably be idealized as two- and three-dimensional geometric shapes. Identify the ways in which the actual shape varies from the idealized geometric model.

3 Model situations with equations and inequalities.

Include situations well described by a linear inequality in two variables or a system of linear inequalities defining a region in the plane.

4 Model situations with common functions.

Include situations well described by linear, quadratic or exponential functions; and situations that can be well described by inverse variation ($f(x) = k/x$). Include identifying a family of functions that models features of a problem, and identifying a particular function of that family and adjusting it to fit by changing parameters. Understand the recursive nature of situations modeled by linear and exponential functions.

5 Model situations using probability and statistics.

Include using simulations to model probabilistic situations; describing the shape of a distribution of values and summarizing a distribution with measures of center and variability; modeling a bivariate relationship using a trend line or a regression line.

6 Interpret the results of applying a model and compare models for a particular situation.

Include realizing that models seldom fit exactly and so there can be error; identifying simple sources of error and being careful not to over-interpret models. Include recognizing that there can be many models that relate to a situation, that they can capture different aspects of the situation, that they can be simpler or more complex, and that they can have a better or worse fit to the situation and the questions being asked.

Shape

Core Concepts · Students understand that:

A Shapes and their parts, attributes, and measurements can be analyzed deductively.*

B Congruence, similarity, and symmetry can be analyzed using transformations.

C Mathematical shapes model the physical world, resulting in practical applications of geometry.

D Right triangles and the Pythagorean theorem are central to geometry and its applications, including trigonometry.

A Coherent Understanding of Shape. From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships. Once understood, those attributes and relationships can be applied in diverse practical situations—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Understanding the attributes of geometric objects often relies on measurement: a circle is a set of points in a plane at a fixed distance from a point; a cube is bounded by six squares of equal area; when two parallel lines are crossed by a transversal, pairs of corresponding angles are congruent.

The concepts of congruence, similarity and symmetry can be united under the concept of geometric transformation. Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. Applying a scale transformation to a geometric figure yields a similar figure. The transformation preserves angle measure, and lengths are related by a constant of proportionality. If the constant of proportionality is one, distances are also preserved (so the transformation is a rigid transformation) and the figures are congruent.

The definitions of sine, cosine and tangent for acute angles are founded on right triangle similarity, and, with the Pythagorean theorem, are fundamental in many practical and theoretical situations.

Connections to Coordinates, Functions and Modeling. The Pythagorean theorem is a key link between geometry, measurement and distance in the coordinate plane. Parameter changes in families of functions can be interpreted as transformations applied to their graphs and those functions, as well as geometric objects in their own right, can be used to model contextual situations.

*In this document, deductive analysis aligns with the notion of adaptive reasoning as defined in *Adding it Up*, and includes empirical exploration, informal justification, and formal proof.

Core Skills · Students can and do:

1 Use multiple geometric properties to solve problems involving geometric figures.

Properties include: measures of interior angles of a triangle sum to 180°; vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; measures of supplementary angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints; and a line tangent to a circle is perpendicular to the radius meeting it.

2 Prove theorems, test conjectures and identify logical errors.

Include theorems establishing the properties in Core Skill 1 and other theorems about angles, parallel and perpendicular lines, similarity and congruence of triangles.

3 Construct and interpret representations of geometric objects.

Include classical construction techniques and construction techniques supported by modern technologies. Include moving between two-dimensional representations and the three-dimensional objects they represent, such as in schematics, assembly instructions, perspective drawings and multiple views.

4 Solve problems involving measurements.

Include measurement (length, angle measure, area, surface area, and volume) of a variety of figures and shapes in two- and three-dimensions. Compute measurements using formulas and by decomposing complex shapes into simpler ones.

5 Solve problems involving similar triangles and scale drawings.

Include computing actual lengths, areas and volumes from a scale drawing and reproducing a scale drawing at a different scale.

6 Apply properties of right triangles and right triangle trigonometry to solve problems.

Include using the Pythagorean theorem and properties of special right triangles, and applying sine, cosine and tangent to determine lengths and angle measures of right triangles. Use right triangles and their properties to solve real-world problems. Limit angle measures to degrees.

Coordinates

Core Concepts · Students understand that:

A Locations in the plane or in space can be specified by pairs or triples of numbers called coordinates.

B Coordinates link algebra with geometry and allow methods in one domain to solve problems in the other.

C The set of solutions to an equation in two variables forms a curve in the coordinate plane—such as a line, parabola, circle—and the solutions to systems of equations correspond to intersections of these curves.

A Coherent Understanding of Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling and proof.

Coordinate geometry is a rich field for exploration. How does a geometric transformation such as a translation or reflection affect the coordinates of points? How is the geometric definition of a circle reflected in its equation?

Adding a third perpendicular axis associates three numbers with locations in three dimensions and extends the use of algebraic techniques to problems involving the three-dimensional world we live in.

Connections to Shape, Quantity, Equations and Functions. Coordinates can be used to reason about shapes. In applications, coordinate values often have units (such as meters and bushels). A one-variable equation of the form $f(x) = g(x)$ may be solved in the coordinate plane by finding intersections of the curves $y = f(x)$ and $y = g(x)$.

Core Skills · Students can and do:

1 Translate fluently between lines in the coordinate plane and their equations.

Include predicting visual features of lines by inspection of their equations, determining the equation of the line through two given points, and determining the equation of the line with a given slope passing through a given point.

2 Identify the correspondence between parameters in common families of equations and the location and appearance of their graphs.

Include common families of equations—the graphs of $Ax + By = C$, $y = mx + b$ and $x = a$ are straight lines; the graphs of $y = a(x - h)^2 + k$ and $y = Ax^2 + Bx + C$ are parabolas; and the graph of $(x - h)^2 + (y - k)^2 = r^2$ is a circle.

3 Use coordinates to solve geometric problems.

Include proving simple theorems algebraically, using coordinates to compute perimeters and areas for triangles and rectangles, finding midpoints of line segments, finding distances between pairs of points and determining when two lines are parallel or perpendicular.

Probability

Core Concepts · Students understand that:

A Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of uncertainty in terms of relative frequency of occurrence.

B The law of large numbers provides the basis for estimating certain probabilities by use of empirical relative frequencies.

C The laws of probability govern the calculation of probabilities of combined events.

D Interpreting probabilities contextually is essential to rational decision-making in situations involving randomness.

A Coherent Understanding of Probability. Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1. This number is generally interpreted as the relative frequency of occurrence of the event over the long run.

The structure of a probability model begins by listing or describing the possible outcomes for a random situation (the sample space) and assigning probabilities based on an assumption about long-run relative frequency. In situations such as flipping a coin, rolling a number cube, or drawing a card, it is reasonable to assume various outcomes are equally likely.

Compound events constructed from these simple ones can be represented by tree diagrams and by frequency or relative frequency tables. The probabilities of compound events can be computed using these representations and by applying the additive and multiplicative laws of probability. Interpreting these probabilities relies on an understanding of independence and conditional probability, approachable through the analysis of two-way tables.

Converting a verbally-stated problem into the symbols and relations of probability requires careful attention to words such as *and*, *or*, *if*, and *all*, and to grammatical constructions that reflect logical connections. This is especially true when applying probability models to real-world problems, where simplifying assumptions are also usually necessary in order to gain at least an approximate solution.

Connections to Statistics and Expressions. Probability is the foundation for drawing valid conclusions from sampling or experimental data. Counting has an advanced connection with Expressions through Pascal's triangle and binomial expansions.

Core Skills · Students can and do:

1 Compute theoretical probabilities by systematically counting points in the sample space.

Make use of symmetry and equally likely outcomes. Include permutation and combination problems as long as small numbers are involved or technology is used, so that formulas are not required.

2 Interpret probabilities of compound events using concepts of independence and conditional probability.

Include reading conditional probabilities from two-way tables.

3 Compute probabilities of compound events.

Make use of the additive and multiplicative laws of probability, tree diagrams and frequency or relative frequency tables in real contexts. Do not emphasize fluency with the related formulas

4 Estimate probabilities empirically.

Include using data from simulations carried out with technology to estimate probabilities.

5 Identify and explain common misconceptions regarding probability.

Include misconceptions about long-run versus short-run behavior of relative frequencies (the law of large numbers). Include attention to the use and misuse of probability in the media, especially in terms of interpreting charts and tables and in the contextual meaning of terms connected to probability, such as 'odds' or 'risk.'

6 Adapt probability models to solve real-world problems.

Include the use of conditional probability to assess subsets of data (e.g., what does the data say about males and females separately). Include the use of independence as a simplifying assumption (e.g., find the probability that two students both contract the disease this year).

Statistics

Core Concepts · Students understand that:

A Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.

B Visual displays and summary statistics condense the information in data sets into usable knowledge.

C Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic.

D The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

A Coherent Understanding of Statistics. Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

Data are gathered, displayed, summarized, examined and interpreted to discover patterns. Data can be summarized by a statistic measuring center, such as mean or median, and a statistic measuring spread, such as interquartile range or standard deviation. Different distributions can be compared numerically using these statistics or visually using plots. Which statistics to compare, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance and this can be evaluated only under the condition of randomness.

In critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were collected, and the analyses employed as well as the data summaries and the conclusions drawn.

Connections to Probability, Functions and Modeling. Valid conclusions about a population depend on designed simulations or other statistical studies using random sampling or assignment and rely on probability for their interpretation. Functional models may be used to approximate data. If the data are approximately linear, the relationship may be modeled with a trend line and the strength and direction of such a relationship may be expressed through a correlation coefficient. Technology facilitates the study of statistics by making it possible to simulate many possible outcomes in a short amount of time, and by generating plots, function models, trend lines and correlation coefficients.

Core Skills · Students can and do:

1 Formulate questions that can be addressed with data. Identify the relevant data, collect and organize it to respond to the question.

Include determining whether a question can best be addressed through a sample survey, randomized experiment or observational study. Include unbiased selection for a sample and randomization of assignment to treatment for an experiment.

2 Use appropriate displays and summary statistics for data.

Include univariate, bivariate, categorical and quantitative data. Include the thoughtful selection of displays and measures of center and spread to summarize data.

3 Interpret data displays and summaries critically; draw conclusions and develop recommendations.

Include paying attention to the context of the data, interpolating or extrapolating judiciously, and examining the effects of extreme values of the data on summary statistics of center and spread. Include data sets that follow a normal distribution. Include observing and interpreting linear trends in bivariate quantitative data.

4 Draw statistical conclusions involving population means or proportions using sample data.

Conclusions should be based on simulations or other informal techniques, rather than formulas.

5 Evaluate reports based on data.

Include looking for bias or flaws in the way the data were gathered or presented, as well as unwarranted conclusions, such as claims that confuse correlation with causation.

Sample of Works Consulted

I. National Reports and Recommendations

A. *Adding it Up: Helping Children Learn Mathematics*. National Council of Teachers of Mathematics, Mathematics Learning Study Committee, 2001.

B. *Crossroads in Mathematics*, 1995 and *Beyond Crossroads, 2006*. American Mathematical Association of Two-Year Colleges (AMATYC).

C. *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. National Council of Teachers of Mathematics, 2006.

D. *Foundations for Success: Final Report of the National Mathematics Advisory Panel* (NMAP), 2008.

E. *Guidelines for Assessment and Instruction in Statistics Education (GAISE) project* http://www.amstat.org/education/gaise/.

F. *Habits of Mind: An Organizing Principle for a Mathematics Curriculum*. Cuoco, A. , Goldenberg, E. P., and Mark, J. (1996). Journal of Mathematical Behavior, 15 (4), 375-402. Last retrieved July 15, 2009, from http://www2.edc.org/CME/showcase/HabitsOfMind.pdf.

G. *How People Learn: Brain, Mind, Experience, and School.* Bransford, J.D., A.L. Brown and R.R. Cocking, eds.. Committee on Developments in the Science of Learning, Commission on Behavioral and Social Sciences and Education, National Research Council, 1999.

H. *Mathematics and Democracy, The Case for Quantitative Literacy*, edited by Lynn Arthur Steen. National Council on Education and the Disciplines, 2001.

I. *The Opportunity Equation: Transforming Mathematics and Science Education for Citizenship and the Global Economy*. The Carnegie Corporation of New York and the Institute for Advanced Study, 2009. Online: http://www.opportunityequation.org/

J. *Principles and Standards for School Mathematics*. National Council of Teachers of Mathematics, 2000.

K. *Quantitative Literacy and Mathematical Competencies*. Niss, Mogen. Last retrieved July 15, 2009, from http://www.maa.org/QI/pgs215_220.pdf

L. *A Research Companion to Principles and Standards for School Mathematics*. National Council of Teachers of Mathematics, 2003.

M. *Focus in High School Mathematics*: *Reasoning and Sense Making*. National Council of Teachers of Mathematics. Reston, VA: NCTM, in press.

N. *What is Mathematics? A Pedagogical Answer to a Philosophical Question.* Harel, G.. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics from the Perspective of Mathematicians*, Mathematical Association of America, 2008.

O. Blum, W., Galbraith, P. L., Henn, H-W. and Niss, M. (Eds) *Applications and Modeling in Mathematics Education*, ICMI Study 14. Amsterdam: Springer.

II. College Readiness

A. ACT College Readiness Benchmarks™ last retrieved July 14, 2009, from http://www.act.org/research/policymakers/pdf/benchmarks.pdf

B. ACT College Readiness Standards™

C. ACT National Curriculum Survey™

D. Adelman, Cliff. *The Toolbox Revisited: Paths to Degree Completion From High School Through College*, 2006. http://www.ed.gov/rschstat/research/pubs/toolboxrevisit/index.html

E. *Advanced Placement Calculus, Statistics and Computer Science Course Descriptions*. *May 2009, May 2010.* College Board, 2008.

F. *Aligning Postsecondary Expectations and High School Practice: The Gap Defined* (Policy Implications of the ACT National Curriculum Survey Results 2005-2006). Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/NCSPolicyBrief.pdf

G. *Condition of Education, 2004: Indicator 30, Top 30 Postsecondary Courses, U.S. Department of Education,* 2004.

H. *Condition of Education, 2007: High School Course-Taking*. U.S. Department of Education, 2007.

I. Conley, D.T. *Knowledge and Skills for University Success*, 2008.

J. Conley, D.T. *Toward a More Comprehensive Conception of College Readiness*, 2007.

K. *Crisis at the Core: Preparing All Students for College and Work.* ACT, Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/crisis_report.pdf

L. Achieve, Inc., Florida Postsecondary Survey, 2008.

M. Golfin, Peggy, et. al. CNA Corporation. *Strengthening Mathematics at the Postsecondary Level: Literature Review and Analysis*, 2005.

N. Camara, W.J., Shaw, E., and Patterson, B. (June 13, 2009). First Year English and Math College Coursework. College Board: New York, NY (Available from authors).

O. CLEP Precalculus Curriculum Survey: Summary of Results. The College Board, 2005.

P. College Board Standards for College Success: Mathematics and Statistics. College Board, 2006. http://www.collegeboard.com/prod_downloads/about/association/academic/mathematics-statistics_cbscs.pdf.

Q. Miller, G.E., Twing, J., and Meyers, J. "Higher Education Readiness Component (HERC) Correlation Study." Austin, TX: Pearson.

R. *On Course for Success: A Close Look at Selected High School Courses That Prepare All Students for College and Work.* ACT. Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/success_report.pdf

S. *Out of Many, One: Towards Rigorous Common Core Standards from the Ground Up*. Achieve, 2008.

T. *Ready for College and Ready for Work: Same or Different?* ACT. Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/ReadinessBrief.pdf

U. *Ready or Not: Creating a High School Diploma That Counts.* American Diploma Project, 2004. http://www.achieve.org/ReadyorNot

V. Rigor at Risk: Reaffirming Quality in the High School Core Curriculum. ACT, Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/rigor_report.pdf

W. *The Forgotten Middle: Ensuring that All Students Are on Target for College and Career Readiness before High School.* ACT, Last retrieved July 14, 2009, from http://www.act.org/research/policymakers/reports/ForgottenMiddle.html

X. Achieve, Inc., Virginia Postsecondary Survey, 2004.

III. Career Readiness

A. ACT Job Skill Comparison Charts

B. Achieve's Mathematics at Work, 2008. http://www.achieve.org/mathatwork

C. *The American Diploma Project Workplace Study*. National Alliance of Business Study, 2002.

D. Carnevale, Anthony and Desrochers, Donna. *Connecting Education Standards and Employment: Course-taking Patterns of Young Workers,* 2002.

http://www.edroundtable.state.in.us/pdf/adp/ADPWorkplaceStudy.pdf

E. Colorado Business Leaders Top Skills, 2006.

F. *Hawai'i Career Ready Study: access to living wage careers from high school*, 2007.

G. *Ready or Not: Creating a High School Diploma That Counts.* American Diploma Project, 2004. http://www.achieve.org/ReadyorNot

H. States' Career Cluster Initiative. *Essential Knowledge and Skill Statements*, 2008.

I. WorkKeys Occupational Profiles™

http://www.act.org/workkeys/profiles/occuprof/index.html

IV. International Documents

STANDARDS [High performing countries/countries of interest]

A. Alberta

Alberta Learning, *Pure Mathematics 10–20–30 (Senior High)*, 2002. (Grades 10-12)

B. Belgium

Department for Educational Development: Core Curriculum. (Grades 9-10 and 11-12) Retrieved June 24, 2009 from http://www.ond.vlaanderen.be/dvo/english/corecurriculum/corecurriculum.htm

C. China

Compulsory mathematics curriculum for senior secondary, as submitted to the APEC consortium. (Grades 10-12) Last retrieved July 13, 2009, from http://www.apecknowledgebank.org/file.aspx?id=1895

D. Chinese Taipei

- Curriculum Guidelines for the Required Subject Math in Senior High Schools, as submitted to the APEC consortium. (Grades 10-11) Last retrieved July 13, 2009 from http://www.apecknowledgebank.org/file.aspx?id=1900
- Curriculum Guidelines for the Elective Subject Math (I) in Senior High Schools, as submitted to the APEC consortium. (Grades 12) Last retrieved July 13, 2009, from http://www.apecknowledgebank.org/file.aspx?id=1901

E. England

Qualifications and Curriculum Authority. *Programme of Study for Key Stage 4, National Curriculum,* 2007. (Grades 10-11)

F. Finland

Finnish National Board of Education. *National Core Curriculum For Upper Secondary Schools 2003*. (Grades 10-12)

G. Hong Kong

Learning Objectives for Key Stage 4. (Grades 10-11)

H. India

- *The National Curriculum Framework,* 2005. (Grades 9-12). National Council of Educational Research and Training. Last retrieved July 13, 2009, from http://ncert.nic.in/html/pdf/schoolcurriculum/framework05/prelims.pdf
- *Secondary School Curriculum, (2010)*. (Grades 9-10). Central Board of Education.
- *Senior School Curriculum, (2010)*. (Grades 11-12). Central Board of Secondary Education.

I. Ireland

- *Junior Certificate Mathematics Syllabus (Higher, Ordinary and Foundation Level)*. (Grades 7-9). National Council for Curriculum and Assessment, 2000.
- The Leaving Certificate *Mathematics Syllabus. (Higher, Ordinary and Foundation Level)*. (Grades 10-12). National Council for Curriculum and Assessment, 2000.

J. Japan

The Courses of Study in Japan, Provisional and Unofficial Translation, 2004, as submitted to the APEC consortium. (Grades 10-12). Last retrieved July 13, 2009, from http://www.apecknowledgebank.org/file.aspx?id=1896.

K. Korea

The Seventh National School Curriculum, as submitted to the APEC consortium. (Grades 7-10). Last retrieved July 13, 2009, from http://www.apecknowledgebank.org/file.aspx?id=1897 and http://www.apecknowledgebank.org/file.aspx?id=1898

L. Singapore

Secondary Mathematics Syllabuses, 2007, as submitted to the APEC consortium. (Grades ≈7-10/11). Last retrieved July 13, 2009, from http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-secondary.pdf.

ASSESSMENTS [Key international assessments with global reach]

A. Program for International Student Assessment (PISA), 2006.

B. Trends in International Mathematics and Science Study (TIMSS), 2007.

C. International Baccalaureate, Mathematics Standard Level, 2006.

D. University of Cambridge International Examinations: General Certificate of Secondary Education in Mathematics, 2009.

E. EdExcel, General Certificate of Secondary Education, Mathematics, 2009.

F. Lundsgaard Hansen, Birthe and Vagn. Reference levels in School Mathematics, Education in Europe, National Presentation, Denmark, 2001. Last retrieved July 15, 2009, from http://www.emis.de/projects/Ref/doc_ems_pdf/EMS_NATIONAL_PRESENTATIONS/EMS_DENMARK.pdf

V. State Documents

A. California State Standards

B. Florida State Standards

C. Georgia State Standards

D. Massachusetts State Standards

E. Mathematics Curriculum Framework Revision Panel [Revised Progress Report]. Massachusetts Department of Elementary and Secondary Education, 2009. Last retrieved July 15, 2009, from http://www.doe.mass.edu/boe/docs/0509/item5_report.pdf.

F. Minnesota State Standards

English Language Arts

K–12

Introduction

The *Standards for English Language Arts K–12* are the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of English language arts (ELA) standards. Its companion document, *Standards for Literacy in History and Science 6–12*, extends the same principle to communication skills in other content areas. The present work, led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by states in their decades-long work on crafting high-quality education standards. The *Standards* also draw upon the most important international models as well as research and input from numerous sources, including scholars, assessment developers, professional organizations, and educators from kindergarten through college. In their design and content, the *Standards* represent a synthesis of the best elements of standards-related work to date and an important advance over that previous work.

As specified by CCSSO and NGA, the *Standards* are (1) research and evidence based, (2) aligned with college and work expectations, (3) rigorous, and (4) internationally benchmarked. A particular standard was to be included in the document only when the best available evidence indicated that its mastery was essential for students to be college and career ready in a twenty-first-century, globally competitive society. As new and better evidence emerges, the *Standards* will be revised accordingly.

The *Standards* are an extension of a prior initiative led by CCSSO and NGA to develop college and career readiness (CCR) standards in reading, writing, and speaking and listening as well as in mathematics. The CCR Reading, Writing, and Speaking and Listening Standards, released in draft form in September 2009, served as a touchstone for the present work. While the format, structure, content, and purpose of that earlier document differ in some ways from this document, the basic aims and concepts are clearly connected. The main difference is that while the earlier CCR document defined a goal toward which education efforts should aim—college and career readiness for all students—the current document describes the progressive development of skills and understandings across the grades necessary for all students to reach that goal. Just as feedback on the September 2009 CCR draft has greatly influenced the design and development of the K–12 standards, so too will the response to the K–12 standards help guide subsequent revisions to the CCR standards. In their final forms, both documents—CCR and K–12—will be tightly aligned and mutually supporting.

While the *Standards* treat college and career readiness for all students as the end point—an ambitious goal in its own right—many students will reach this point before the end of high school. For them, advanced work in literature, composition, language, history, science, and so on should be available. It is beyond the scope of the *Standards* to describe what such advanced work should consist of, but it needs to provide the next logical step up from the college and career readiness baseline established here.

As a natural outgrowth of meeting the charge to define college and career readiness, the *Standards* also lay out a vision of what it means to be a literate person in the twenty-first century. Indeed, the skills and understandings students must demonstrate have broad applicability outside of the classroom or workplace. The *Standards* insist upon the sort of close, attentive reading that is at the heart of understanding and appreciating the aesthetics of literature. They require the sort of critical reading that is necessary to sift carefully through the staggering amount of information available today in print and online. They demand the sort of wide, deep, and thoughtful engagement with high-quality literary and informational text that builds knowledge, enlarges experience, and broadens world views. They mandate the sort of cogent reasoning and use of evidence that is essential to both private deliberation and responsible citizenship in a democratic republic. In short, they promote the development of skills in reading, writing, speaking, and listening that are the foundation for any creative and purposeful expression in language.

Key design considerations

A blend of cross-cutting and specific standards

The Reading, Writing, and Speaking and Listening strands include two levels of standards. The cross-cutting Core Standards are the same across the two *Standards* documents, their commonality emphasizing the broad responsibility within the school for meeting the standards and also facilitating schoolwide professional development. Then there are specific Standards that are unique to a given content area, which respects the particular demands of reading, writing, speaking, and listening in ELA and in other disciplines.

A focus on results rather than means

The *Standards* define what all students must learn, not everything that teachers are allowed to teach. By focusing on required achievements, the *Standards* leave room for teachers, curriculum developers, and states to determine how those goals should be reached and what additional topics should be addressed. The *Standards* require, for example, that all students be able to produce writing in a variety of situations, including those that allow time for revision. The *Standards* do not, however, specify a particular writing process that students must use (although certain elements common to process-writing approaches, particularly revision, are embedded in the requirements). Teachers are thus freed—and obligated—to provide students with whatever tools and knowledge their professional judgment and experience identify as most helpful for those assignments that allow for multiple drafts. Similarly, the *Standards*, with their emphasis on observable outcomes, do not enumerate various metacognitive strategies that students may need to use to monitor and direct their thinking and learning.

Shared responsibility for literacy

The *Standards for English Language Arts K–12* and the *Standards for Literacy in History and Science 6–12* together establish the requirement that instruction in reading, writing, speaking, listening, and language use be a shared responsibility. The *Standards* present reading instruction in K–5 as fully integrative, including a rich blend of narratives, drama, poetry, and informational text. ELA-specific instruction in grades 6 and above includes fiction, poetry, and drama but also a particular form of informational text: literary exposition and argument (e.g., speeches, essays, and historical documents with significant cultural importance and literary merit). Teachers in other content areas must use their unique disciplinary expertise to help students meet the particular challenges of reading, writing, speaking, listening, and language use in their respective field. Progress toward college and career readiness and building a rich knowledge base require that at least half of the reading students do must focus on history, science and related disciplines. This distributed approach honors the unique place of English language arts instruction in literacy development while ensuring that students have communication skills tailored to the demands of other disciplines. It also reflects the reality that students must communicate effectively in a wide range of disciplines, not just ELA.

Grade bands to describe growth, grades to focus instruction

Evidence consulted in creating the *Standards* suggests that beyond the earliest grades, major developments in students' literacy skills typically occur across spans of grades rather than within individual grades. This document stays true to that evidence by organizing standards after grade 3 into multiyear bands (grades 4–5, 6–8, 9–10, and 11–CCR). At the same time, the work of educating students does proceed on a day-to-day, year-to-year basis. Any standards document must therefore provide guidance to educators on what each year's instruction and assessment should look like. To make the grade specific focuses for instruction clear, after the descriptions of the standards in each area of ELA, we provide a one page summary of the grade specific focuses for each grade from fourth grade onwards, including how the grade specific focus in each area relates to the grade band requirements. The Standards offer that focus through several grade-specific elements:

- Single-grade standards in many areas of kindergarten and grades 1, 2, and 3
- Text complexity expectations in Reading, beginning at grade 2
- Areas of focus in Writing, beginning at grade 4

- Areas of focus in the Conventions section of Language Development, beginning at grade 4

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Research and media skills integrated into the standards as a whole

To be ready to meet the challenges of the twenty-first century, students need a mix of the communication skills that have served literate people for millennia and new competencies necessary in an information- and media-saturated world. To be ready for college, workforce training, and life in a technological society, students need the ability to gather, comprehend, evaluate, synthesize, report on, and create a high volume and extensive range of print and nonprint texts in media forms old and new. Just as the need to research and to consume and produce media are embedded into every element of today's curriculum, so too are the associated skills and understandings embedded throughout the *Standards* rather than treated separately. Web links to sample media texts are included selectively among the reading text exemplars in Appendix B to reinforce the point that print and online materials can be used together instructionally to enhance students' understanding.

An integrated model of English language arts

Although the *Standards* divide the English language arts into Reading, Writing, Speaking and Listening, and Language Development strands for conceptual purposes, the processes of communication are in theory and practice an undivided whole. As illustrated in the graphic that introduces each grade or grade band and as embodied in the content of the standards themselves, reading, writing, speaking, listening, and language development are tightly interrelated and often reciprocal.

Central features of the document

Reading and Literature: Text complexity and the growth of comprehension

As students advance through the grades, they must be able to handle independently texts of steadily increasing complexity and be able to gain more from what they read. Beginning formally at grade 2, the *Standards* specify what proportion of texts students read should be within grade band and, at some grades, above grade band. (Additional material in Appendix A of the *Standards* defines and explains text complexity in more detail.) Whatever texts they are reading, students must also show a steadily increasing ability to discern more from and make fuller use of text. This means, for example, finding and making an increasing number of connections among ideas and between texts; considering a wider range of textual evidence; and becoming more sensitive to inconsistencies, ambiguities, and poor reasoning in texts. The *Standards* place growing demands on students' comprehension at each higher grade or grade band to ensure that all students are college- and career-ready readers no later than the end of high school.

Writing and Research: Text types, grade-level focuses, and research

While some writing skills, such as the ability to reflect audience, purpose, and task in what one writes, are important for many types of writing, others are more properly part of writing in specific text types: narrative, informative and explanatory text, and argument. Beginning at grade 4, the *Standards* specify the sorts of writing over extended and shorter timeframes that students in each grade must be able to produce in response to sources. Although conducting research calls upon reading, speaking, listening, and language skills, writing is typically central to analyzing information and presenting findings. The *Standards* pair writing and research to signal that close connection.

Speaking and Listening: Flexible communication

Including but not limited to skills necessary for formal presentations, the Speaking and Listening strand requires students to develop a range of broadly useful oral communication and interpersonal skills: listening attentively, participating productively, exchanging information, and speaking effectively. Students must learn to sift through and evaluate multiple points of view; listen thoughtfully in order to build on and constructively question the ideas of others while contributing their own ideas; and, where appropriate, reach agreement and common goals through teamwork.

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Language Development: Conventions and vocabulary

The Conventions standards in the Language Development strand include the essential "rules" of formal written and spoken English, but they also approach language as a matter of craft and informed choice among alternatives. Thus, standards pertaining to grammar and usage, mechanics, and the fundamentals of language and writing are accompanied by standards on word choice and style. The Vocabulary standards focus both on understanding words and their nuances and on acquiring new words through conversation and reading and by being taught them directly. Rather than require that students use one particular skill or another to determine a word's meaning, the Vocabulary standards insist only that students get the proper meaning, with the means (context, word analysis, and so on) to be chosen flexibly based on the situation.

Appendices

The *Standards* include a range of supporting materials that help explain and enrich the main document:

- Appendix A contains a model of text complexity, including both qualitative and quantitative measures of how easy or hard a text is to read, as well as supplementary statements about instruction in writing, language conventions, and vocabulary
- Appendix B consists of text exemplars at all grades/bands to illustrate appropriate complexity and quality in the text types required by the Reading standards
- Appendix C consists of annotated writing samples to show how grade- or grade-band-appropriate writing embodies the relevant Writing standards

January 2010

Student Practices in Reading, Writing, Speaking, Listening, and Language Use

The following Student Practices in reading, writing, speaking, listening, and language use undergird and help unify the rest of the *Standards*. The Student Practices are not themselves standards: every idea introduced here is subsequently represented in one or more places within the larger document. They are, rather, the "premises"—broad statements about the nature of college and career readiness in reading, writing, speaking, listening, and language use—that underlie the individual standards and cut across the various sections of the document.

* * *

As students progress toward being college and career ready, they exhibit with increasing fullness and regularity the following capacities in their reading, writing, speaking, listening, and language use:

1. They demonstrate independence as readers, writers, speakers, listeners, and language users.

Students can, without significant scaffolding or support, comprehend and evaluate complex text across a range of types and disciplines, and they can construct effective arguments and clearly convey intricate or multifaceted information. Likewise, students are independently able to discern a speaker's key points as well as ask questions, build on others' ideas, and articulate their own ideas. They apply language conventions without prompting. On their own, they determine the meaning of words in context and acquire and use new words.

2. They build strong content knowledge.

Students build a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They demonstrate their ability to become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and the discipline-specific expertise needed to comprehend subject matter and solve problems in different fields. They refine their knowledge and share it through substantive writing and speaking.

3. They respond to the varying demands of audience, task, purpose, and discipline.

Students consider their reading, writing, speaking, listening, and language use in relation to the contextual factors of audience, task, purpose, and discipline. They appreciate nuances, such as how the composition and familiarity of the audience should affect tone and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in the sciences).

4. They comprehend as well as critique.

Students are engaged and open-minded—but skeptical—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and assess the veracity of claims.

5. They privilege evidence.

Students cite specific evidence when offering an oral or written interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

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6. They care about precision.

Students are mindful of the impact of specific words and details, and they consider what would be achieved by different choices. Students pay especially close attention when precision matters most, such as in the case of reviewing significant data, making important distinctions, or analyzing a key moment in the action of a play or novel.

7. They craft and look for structure.

Students attend to structure when organizing their own writing and speaking as well as when seeking to understand the work of others. They understand and make use of the ways of presenting information typical of different disciplines. They observe, for example, how authors of literary works craft the structure to unfold events and depict the setting.

8. They use technology strategically and capably.

Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

9. They come to understand other perspectives and cultures.

Students appreciate that the twenty-first-century classroom and workplace are diverse settings in which people from often widely divergent backgrounds must learn and work together. They actively seek to understand other perspectives and cultures through reading and listening. They do not simply adopt other points of view as their own but rather evaluate them critically and constructively. Literature can play a special role in expanding students' horizons in this way: through reading great classic and contemporary works, students can vicariously inhabit worlds and experiences much different than their own.

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English Language Learners

The *Standards* articulate rigorous grade-level expectations in the areas of speaking, listening, reading and writing to prepare students to be college and career ready. English language learners (ELLs) must be held to the same high standards expected of students who are already proficient in English. However, because these students are acquiring English language proficiency and content area knowledge concurrently, some students will require additional time and all will require appropriate instructional support and aligned assessments.

ELLs are a heterogeneous group with differences in ethnic background, first language, socio-economic status, quality of prior schooling, and levels of English language proficiency. Effectively educating these students requires adjusting instruction and assessment in ways that consider these factors. For example ELLs who are literate in a first language that shares cognates with English can apply first-language vocabulary knowledge when reading in English; likewise ELLs with high levels of schooling can bring to bear conceptual knowledge developed in their first language when reading in a second language. On the other hand, ELLs with limited or interrupted schooling will need to acquire background knowledge prerequisite to educational tasks at hand. As they become acculturated to US schools, ELLs who are newcomers will need sufficiently scaffolded instruction and assessments to make sense of content delivered in a second language and display this content knowledge.

While some ELLs are economically and educationally advantaged, this is not the case for many of these students. Moreover, once in the U.S., the majority of ELLs attend high poverty schools with high percentages of other ELLs. These schools often lack the resources and capacity needed to help ELLs reach high academic standards. However, schools and districts can be assisted in providing a positive learning environment that capitalizes on the linguistic and cultural diversity of the student body.

To help ELLs meet high academic standards in reading, writing, speaking, listening and language use it is essential that ELLs have access to:

- The requisite coursework to prepare them for post-secondary education or the workplace;
- Coursework that is made comprehensible for students learning content in a second language, through specific pedagogical techniques and additional resources;
- Teachers, as well as school-level and district personnel, who are well prepared and qualified to support English-language learners;
- Well designed opportunities for classroom discourse and interaction to enable ELLs to develop communicative strengths in language arts
- Speakers of English who know the language well enough to provide the ELLs with models and support; and
- Ongoing assessment and feedback to guide learning.

It is also worth noting that instruction for these students is additionally guided by language proficiency standards that language arts teachers can use in conjunction with the English language arts standards to help ELLs become fully proficient and literate in English.

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Access for Students with Disabilities

The *Standards* articulate rigorous expectations in the areas of reading, writing, speaking, listening, and language use in order to prepare students to be college and career ready. These standards identify the knowledge and skills students must acquire in order to be successful. Research shows that students with disabilities are capable of high levels of learning and should not be limited by low expectations and watered down curriculum. The vast majority of this population of students, including students with intellectual impairments,[1] *can* achieve proficiency when they receive high-level instruction and accommodations. It is imperative that these highly capable students—regardless of their disability—are held to the same expectations articulated in the Core Standards as other students.

However, *how* these high standards are taught is of the utmost importance in reaching students with special needs. When acquiring the knowledge and skills represented in the Core Standards, students with disabilities may need accommodations[2] or—in exceptional cases—modified goals, incorporated in an individualized education program (IEP),[3] to help them access information or demonstrate their knowledge. In instances when a standard asks students to perform actions they are physically incapable of, students will need to be presented with alternative options to demonstrate similar knowledge and skills within the range of their abilities. Accommodations based on individual needs allow students of all disability levels to learn within the framework of the *Standards*.

Meeting English Language Arts (ELA) Standards

Reading, writing, speaking, listening, and language use standards—given the nature of the standards themselves—often require accommodations for students with disabilities. For example, a standard that calls for "listening" should be interpreted to include reading sign language. "Speaking" should be read broadly to include "communication" or "self-expression." "Reading" should allow for students' use of Braille, screen reader technology, or other assistive devices to demonstrate comprehension skills. In a similar vein, "writing" should not preclude the use of a scribe, computer, or speech-to-text technology. With appropriate accommodations and support, students with all levels of disabilities can participate in the general education curriculum and achieve grade-level proficiency with regard to the ELA content and skills articulated in the *Standards*.

In short, while the *Standards* set and retain high expectations for all students, they may need to be translated and occasionally modified to apply appropriately to students with disabilities, including all levels of intellectual impairment. Promoting a culture of high expectations for all students is a fundamental goal of the *Standards*. Achieving this goal requires the inclusion of students with disabilities.

[1] Less than two percent of the population of all students and less than 20% of the population of students with disabilities.

[2] See the Council of Chief State School Officers, (2003). *Training District and State Personnel on Accommodations: A Study of State Practices, Challenges, and Resources* at http://www.ccsso.org/publications/details.cfm?PublicationID=221 for further explanation and evidence around accommodations.

[3] According to the Individuals with Disabilities Act (IDEA), an IEP includes appropriate accommodations that are necessary to measure the individual achievement and functional performance of a child.

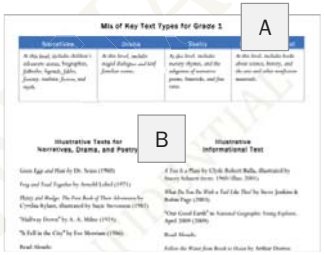
How to Read This Document

The *Standards* are divided into an ELA-specific document (*Standards for English Language Arts K–12*) and a literacy document for history and science (*Standards for Literacy in History and Science 6–12*). The ELA document includes standards for and examples of history and science reading in K–5.

The ELA-specific document is organized by grade (kindergarten and grades 1, 2, and 3) and grade band (grades 4–5, 6–8, 9–10, and 11–CCR). The *Standards for Literacy in History and Science* are organized by grade band (grades 4–5, 6–8, 9–10, and 11–CCR). Each grade/band is divided into strands—Reading, Writing, Speaking and Listening, and Language Development.

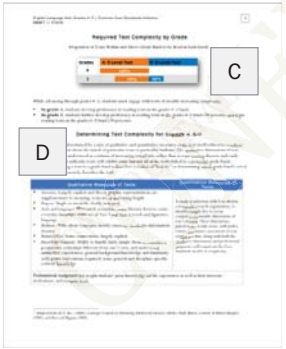
While all strands contain standards statements, each strand also has its own specific features.

Reading and Literature (ELA)/Reading (History/Science)

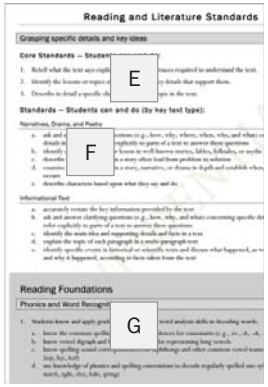


Kindergarten and grade 1 begin with the **mix of key text types (A)**, which identifies the genres and subgenres of reading material appropriate for each grade. This is followed by a list of **illustrative texts (B)** in the key text types. This list is suggestive of the sorts of texts appropriate for the grade in terms of complexity and quality; excerpts appear in Appendix B.

Grades 2, 3, 4–5, 6–8, 9–10, and 11–CCR include this information immediately after a graphic specifying **required text complexity by grade (C)**—in brief, the proportion of texts within and above grade band that students must read each year. (For example, 70% of the texts that grade 3 students read should come from the grades 2–3 text complexity band, while the other 30% should come from the grades 4–5 band.) An overview of the method for **determining text complexity (D)** in the particular grade band follows. (A fuller treatment is provided in Appendix A.)

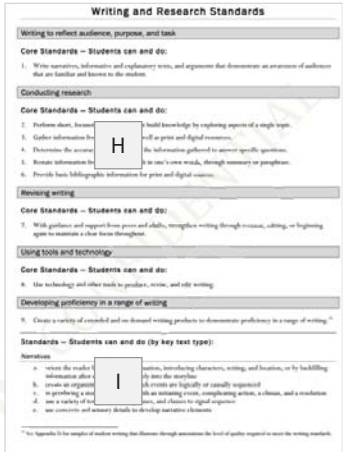


All grades/bands organize standards under a number of boxed subheadings (e.g., "Grasping specific details and key ideas"). The standards at all grade levels are divided into cross-cutting **Core Standards (E)**, which are numbered and applicable to many types of reading, and more specific **Standards (F)**, which are lettered and organized by text type (e.g., "Narratives, Drama, and Poetry"). Kindergarten and grades 1, 2, and 3 also include boxed sections of **reading foundations (G)**, which enumerate basic concepts of print and other foundational skills in reading that very young students must acquire.



Writing and Research

Cross-cutting **Core Standards (H)**, which are numbered and apply to many types of writing, are organized under a number of boxed subheadings (e.g., "Writing to reflect audience, purpose, and task"). **Standards (I)** specific to writing in particular text types—narrative, informative and explanatory text, and argument—are lettered and follow in a separate section. In kindergarten through grade 5, writing arguments takes the form of opinion writing, hence the parenthetical notation next to "Arguments" in those grades/bands.


Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards – Students can and do:

Conducting research

Core Standards – Students can and do:

H

Revising writing

Core Standards – Students can and do:

Using tools and technology

Core Standards – Students can and do:

Developing proficiency in a range of writing

Standards – Students can and do (by key text type):

Narratives

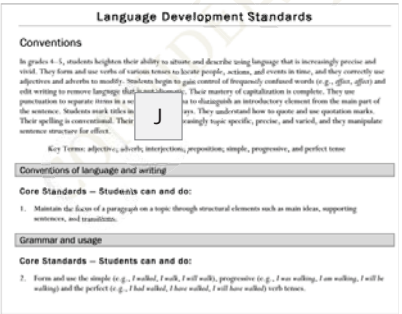
I

Speaking and Listening

Cross-cutting **Core Standards**, which are numbered and apply to speaking and listening in many situations, are grouped with lettered **Standards**, which set requirements for speaking and listening in key communication (e.g., "Presentation of Ideas and Information").

Language Development

This strand is organized differently in the ELA and history/science documents. The ELA strand comprises two full sections: Conventions and Vocabulary. Each section includes numbered **Core Standards** organized under a number of boxed subheadings (e.g., "Mechanics") and introduced by a brief summative paragraph and list of **key terms (J)** intended to be taught explicitly in grade-appropriate ways. ELA Vocabulary consists of three subsections—determining the meaning of words, understanding the nuances of words, and acquiring vocabulary—under each of which numbered Core Standards appear. Language Development in the history/science document consists of the summative paragraphs for Conventions and all of the Vocabulary section found in ELA except for nuances in word meaning.


Language Development Standards

Conventions

J

Key Terms: adjectives, adverb, interjections, preposition, simple, progressive, and perfect tense

Conventions of language and writing

Core Standards – Students can and do:

Grammar and usage

Core Standards – Students can and do:

Focus for Instruction

Beginning with grade 4, a page summarizing grade-level responsibility, including grade-specific areas of focus, is provided to help distinguish responsibilities within multiyear bands.

English Language Arts

Kindergarten

Mix of Key Text Types for Kindergarten

Narratives	Drama	Poetry	Informational Text
At this level, includes children's adventure stories, biographies, folktales, legends, fables, fantasy, realistic fiction, and myth.	*At this level, includes staged dialogue and brief familiar scenes.*	*At this level, includes nursery rhymes and the subgenres of narrative poems, limericks, and free verse.*	*At this level, includes books about science, history, and the arts and other nonfiction materials.*

Illustrative Texts for Narratives, Drama, and Poetry[4]

Are You My Mother? by P.D. Eastman (1960)

Put Me in the Zoo by Robert Lopshire (1960)

"Mix a Pancake" by Christina Rosetti (1986)

Wouldn't You?" by John Ciardi (1986)

Illustrative Informational Texts

My Five Senses by Aliki (1962)

Read Alouds:

Amazing Whales! by Sarah L. Thomson (2005)

The Year at Maple Hill Farm by Alice and Martin Provensen (1978)

[4] See Appendix B for other texts illustrative of Kindergarten-Grade 1 text complexity. This list includes read-alouds.

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Reading and Literature Standards[5]

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Retell key details and information drawn from the text.
2. Explain the subject of the text or the problem the characters face.
3. Answer questions about characters and events that take place in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. retell the beginnings, middles, and endings of stories
b. ask and answer questions about details of a text
c. identify the problems that characters face in a story and the lessons learned
d. identify the feelings of characters and the reasons for their actions
e. differentiate between realistic and fantastical elements within a story

Informational Text

a. restate key information (e.g., events, subject, ideas) from a text
b. ask and answer questions about details of a text

Reading Foundations

Print Concepts

1. Students demonstrate understanding of the organization and basic features of print.
   a. identify basic features and conventions of books and other written texts (e.g., front cover, back cover, title, author)
   b. understand that print is left to right, top to bottom, and page by page
   c. understand that words are separated by spaces in print
   d. recognize and name all upper- and lowercase letters of the alphabet

Linguistic Awareness

2. Students gain awareness of spoken words, syllables, and phonemes.
   a. recognize, recite, and produce rhyming words
   b. count, pronounce, blend, and segment syllables in spoken words
   c. recognize, blend, and segment onset and rimes of spoken words [*/g/ - /oat/; /bl/ - /ock/)*
   d. count or place tokens for individual words in spoken phrases or simple sentences
   e. orally blend and segment individual phonemes in simple, one-syllable words
   f. demonstrate phonemic awareness by isolating and pronouncing the initial and final phonemes (sounds) in three-phoneme /CVC/ words without consonant blends (e.g., */road/*, */save/*, */ham/*)
   g. add or substitute individual phonemes in simple, one-syllable words to make new words (e.g., */at/ → /sat/ → /mat/ → /map/)*

[5] The expectation is that students can fulfill these standards with texts they read independently as well as texts that are read aloud to them.

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Observing craft and structure

Core Standards — Students can and do:

4. Identify the meanings of words and phrases as they are used in the text.
5. Identify important parts or sections of texts.
6. Compare and contrast characters or events from different stories.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. identify words and phrases that suggest feelings or appeal to the senses
b. identify similarities in beginning and ending sounds of words in children's poems and songs
c. identify parts of a story and parts of a poem as well as sections of informational picture books and tell how they are different
d. identify common characteristics of folktales and fairy tales, including their use of rhyme, rhythm, and repetition
e. participate (e.g., react, speculate, read along, act out) when familiar texts are read aloud
f. compare and contrast characters or events from different stories written by the same author or written about similar subjects

Informational Text

a. identify basic text features and what they mean, including titles and subtitles, table of contents, and chapters

Reading Foundations, continued

Phonics and Word Recognition

3. Students know and apply grade-level phonics and word analysis skills in decoding words.
   a. demonstrate basic knowledge of letter-sound correspondences by producing the primary or most frequent sound for each consonant
   b. identify which letters represent the five major vowels and know the long and short sound of each
   c. blend letter-sounds to decode short-vowel CVC words (e.g., *cat*, *mop*, *sun*)
   d. read at least 25 very high frequency words by sight (e.g., *of*, *to*, *he*, *she*, *is*, *do*, *does*)

Developing Fluency

4. Students read with sufficient accuracy and fluency to support comprehension.
   a. read rebus and preprimer texts with purpose and understanding
   b. demonstrate increased accuracy and fluency on successive readings of a text

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Use text illustrations to predict or confirm what the text is about.
8. Identify words in a text that link ideas and events together.
9. Identify who is telling a story or providing information in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. use pictures, illustrations, and context to make predictions about and confirm story content
b. identify words in a story that link events together (e.g., *first/second*, *then*, *next*, *before/after*, *later*, *finally*)
c. identify who is the speaker in a story or poem

Informational Text

a. identify words that link ideas together (e.g., *also*, *in addition*, *for example*, *but*)
b. identify the author and sources of information when provided by the text

Developing habits for reading text

Core Standards — Students can and do:

10. Begin to read independently and/or with a partner, sustaining effort necessary to build understanding.

Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and opinions that communicate to a familiar, known audience.

Conducting research

Core Standards — Students can and do:

2. Gather information from experiences or provided text sources.

Revising writing

Core Standards — Students can and do:

3. With specific guidance, add details to strengthen writing through revision.

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Using tools and technology

Core Standards — Students can and do:

4. Gain familiarity with technology and other tools to produce, revise, and edit writing.

Standards — Students can and do (by key text type):[6]

Narratives

a. establish a situation in time and/or place
b. recount several loosely linked actions in a short, familiar event, controlling for chronological order
c. provide a reaction to what happened

Informative and Explanatory Texts

a. establish the topic in a title or first sentence
b. supply facts and information relevant to the topic

Arguments (opinions)

a. introduce the topic directly, or use the title of a book when writing about a text
b. express preferences or opinions (e.g., *My favorite book is . . .*) relevant to the topic
c. provide a reason for preference or opinion (e.g., *It reminded me of when I met my friend Carlos*)
d. use linking words that express causality (e.g., *I like . . . because . . .*)

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively in group activities requiring speaking and listening.
2. Listen closely to and sustain attention on texts read aloud as well as other sources of information presented orally, visually, or multi-modally and confirm understanding by restating the information and answering pertinent questions.

Standards — Students can and do (by key communication type):

Classroom discussions and participating productively

a. initiate and participate in conversations with peers and adults through multiple exchanges, attending to the comments of others
b. confirm understanding by restating information or answering questions about what has been discussed
c. ask questions to get information, ask for help, or clarify something that is not understood
d. follow norms for conversation, such as listening to others and taking turns to speak

Exchanging information and speaking effectively

Core Standards — Students can and do:

[6] See Appendix D for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

3. Share experiences and ideas that demonstrate an awareness of their listeners.
4. Speak audibly and clearly.

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. describe people, places, things, and events with relevant facts and examples
b. recite or read aloud poems, rhymes, songs, and stories, speaking clearly at an understandable pace

Language Development Standards

Conventions

In kindergarten, students learn to form letters and words in print and to relate sounds (phonemes) to one or more letters. They understand the notion of a sentence, that a sentence performs one of a few basic functions (make a statement, ask a question, or issue a command), and that end punctuation can signal the sentence's function or intensity. With prompting and assistance, they form and expand basic sentences in order to express thoughts, beginning the sentence with a capital letter. Students have a sense of what a noun is, of what singular and plural nouns are, and of how plural nouns are often formed. They also know how to use the most frequently occurring prepositions.

Key Terms: exclamation point, capital/uppercase and lowercase letter, singular and plural noun, period, punctuation, question mark, sentence

Conventions of language and writing

Core Standards — Students can and do:

1. Print all upper- and lowercase letters.
2. Write a letter or letters for each consonant and short-vowel sound (phoneme).

Grammar and usage

Core Standards — Students can and do:

3. Produce and expand complete sentences in shared writing and language activities.
4. Use and understand question words (e.g., *what*, *where*, *when*, *who*, *which*, *how*).
5. Form regular plural nouns by adding /s/ or /es/ (e.g., *dog*, *dogs*; *wish*, *wishes*; *baby*, *babies*).
6. Demonstrate understanding of the most frequently occurring prepositions (e.g., *to/from*, *in/out*, *on/off*, *for*, *of*, *by*, *with*).

Mechanics

Core Standards — Students can and do:

7. Capitalize the first word in a sentence and the pronoun *I*.
8. Identify end punctuation, including periods, question marks, and exclamation points.

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9. Spell simple words phonetically using knowledge of sound-letter relationships.

Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In kindergarten, students learn about words in terms of like versus unlike and "similar but not quite the same," using objects and movements as aids. They learn to use descriptive language to distinguish one object from another and order and position words to describe sequences and spatial relationships. They acquire new words through interactive language use, including informal talk, discussion, listening to and responding to texts read aloud as well as by being taught the words directly.

Determining the meaning of words

Core Standards — Students can and do:

1. Sort common objects into categories (e.g., shapes, foods).

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

2. Act out the meaning of verbs describing the same general action (e.g., *walk*, *march*, *strut*, *prance*) to gain a sense of their different meanings.
3. Demonstrate understanding of common adjectives by relating them to their opposites (antonyms).
4. Use common adjectives to distinguish objects (e.g., the *small blue* square, the *shy white* rabbit).

Acquiring vocabulary

Core Standards — Students can and do:

5. Demonstrate meaning of new vocabulary taught directly and gained through conversations and hearing texts read aloud.
6. Demonstrate understanding of words that express order and position (e.g., *first*, *middle*, *last*; *before*, *after*; *under*, *over*).

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English Language Arts

Grade 1

Mix of Key Text Types for Grade 1

Narratives	Drama	Poetry	Informational Text
At this level, includes children's adventure stories, biographies, folktales, legends, fables, fantasy, realistic fiction, and myth.	*At this level, includes staged dialogue and brief, familiar scenes.*	*At this level, includes nursery rhymes and the subgenres of narrative poems, limericks, and free verse.*	*At this level, includes books about science, history, and the arts and other nonfiction materials.*

Illustrative Texts for Narratives, Drama, and Poetry[7]

Green Eggs and Ham by Dr. Seuss (1960)

Frog and Toad Together by Arnold Lobel (1971)

Henry and Mudge: The First Book of Their Adventures by Cynthia Rylant, illustrated by Suçie Stevenson (1987)

"Halfway Down" by A. A. Milne (1924)

"It Fell in the City" by Eve Merriam (1986)

Read alouds:

Little House in the Big Woods by Laura Ingalls Wilder, illustrated by Garth Williams (1932)

Zin! Zin! Zin! a Violin by Lloyd Moss, illustrated by Marjorie Priceman (1995)

Illustrative Informational Texts

A Tree Is a Plant by Clyde Robert Bulla, illustrated by Stacey Schuett (text: 1960/illus: 2001)

What Do You Do With a Tail Like This? by Steve Jenkins & Robin Page (2003)

"Our Good Earth" in *National Geographic Young Explorer*, April (2009)

Read alouds:

Follow the Water from Brook to Ocean by Arthur Dorros (1991)

Living Sunlight: How Plants Bring the Earth to Life by Molly Bang & Penny Chisholm, illustrated by Molly Bang (2009)

[7] See Appendix C for other texts illustrative of Kindergarten-Grade 1 text complexity. This list includes read-alouds.

Reading and Literature Standards[8]

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Retell key details and information drawn from the text.
2. Explain the subject of the text or the problem the characters face.
3. Answer questions about characters and events that take place in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. retell the beginnings, middles, and endings of stories
b. ask and answer questions about details of a text
c. identify the problems that characters face in a story and the lessons learned
d. identify the feelings of characters and the reasons for their actions
e. differentiate between realistic and fantastical elements within a story

Informational Text

a. restate key information (e.g., events, subject, ideas) from a text
b. ask and answer questions about details of a text

Reading Foundations

Linguistic Awareness

1. Students gain awareness of spoken words, syllables, and phonemes.
   a. aurally distinguish long from short vowel sounds in spoken single-syllable words (e.g., /*tap*/ vs. /*tape*/; /*sock*/ vs. /*soak*/; /*sit*/ vs. /*sight*/)
   b. produce single-syllable words by orally blending phonemes, including consonant blends (e.g., /*cats*/, /*black*/, /*blast*/)
   c. isolate and pronounce initial, medial vowel, and final phonemes (sounds) in single-syllable words (e.g., ***f**ast*, *f**a**st*, *fa**s**t*)
   d. orally segment single-syllable words into their complete sequence of individual phonemes

[8] The expectation is that students can fulfill these standards with texts they read independently as well as texts that are read aloud to them.

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Observing craft and structure

Core Standards — Students can and do:

4. Identify the meanings of words and phrases as they are used in the text.
5. Identify important parts or sections of texts.
6. Compare and contrast characters or events from different stories.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. identify words and phrases that suggest feelings or appeal to the senses
b. identify similarities in beginning and ending sounds of words in children's poems and songs
c. identify parts of a story and parts of a poem as well as sections of informational picture books and tell how they are different
d. identify common characteristics of folktales and fairy tales, including their use of rhyme, rhythm, and repetition
e. participate (e.g., react, speculate, read along, act out) when familiar texts are read aloud
f. compare and contrast characters or events from different stories written by the same author or written about similar subjects

Informational Text

a. identify basic text features and what they mean, including titles and subtitles, table of contents, and chapters

Reading Foundations, continued

Phonics and Word Recognition

2. Students know and apply grade-level phonics and word analysis skills in decoding words.

a. know the common spelling-sound correspondences for consonants (e.g., *wr-*, *sh*, *-ck*, *-ll*)
b. know vowel digraph and final-*e* conventions for representing long vowels
c. know spelling-sound correspondences for diphthongs and other common vowel teams (e.g., *loud*, *cow*, *look*, *loop*, *boy*, *boil*)
d. use knowledge of phonics and spelling conventions to decode regularly spelled one-syllable words (e.g., *sick*, *march*, *sight*, *slice*, *bake*, *spring*)
e. understand that every syllable must have a vowel sound and use that knowledge to determine the number of syllables in a word
f. decode two-syllable words following basic patterns (e.g., *rabbit*, *magnet*, *napkin*, *pickle*, *butter*)
g. read words with inflectional endings (e.g., *-s*, *-es*, *ies*, *-ed*, *ied*, *-ing*, *-er*, *-est*)
h. use phonics to decode visually new words when reading
i. recognize grade-appropriate, irregularly spelled words by sight

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Use text illustrations to predict or confirm what the text is about.
8. Identify words in a text that link ideas and events together.
9. Identify who is telling a story or providing information in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. use pictures, illustrations, and context to make predictions about and confirm story content
b. identify words in a story that link events together (e.g., *first/second*, *then*, *next*, *before/after*, *later*, *finally*)
c. identify who is the speaker in a story or poem

Informational Text

a. identify words that link ideas together (e.g., *also*, *in addition*, *for example*, *but*)
b. identify the author and sources of information when provided by the text

Developing habits for reading text

Core Standards — Students can and do:

10. Begin to read independently and/or with a partner, sustaining effort necessary to build understanding.

Reading Foundations, continued

Developing Fluency

3. Students read with sufficient accuracy and fluency to support comprehension.

a. demonstrate increased accuracy, fluency, and expression on successive readings of a text
b. use context to confirm or self-correct word recognition and understanding, rereading as necessary
c. read aloud, alone, or with a partner at least 15 minutes each day, in school or out

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Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and opinions that communicate to a familiar, known audience.

Conducting research

Core Standards — Students can and do:

2. Gather information from experiences or provided text sources.

Revising writing

Core Standards — Students can and do:

3. With specific guidance, add details to strengthen writing through revision.

Using tools and technology

Core Standards — Students can and do:

4. Gain familiarity with technology and other tools to produce, revise, and edit writing.

Standards — Students can and do (by key text type):[9]

Narratives

a. establish a situation in time and/or place that is appropriate for the sequence of events to follow
b. develop appropriately sequenced actions within one or more events using linking words, phrases, or clauses to signal chronological ordering
c. provide a reaction to what happened
d. include dialogue if appropriate, and some details
e. provide a sense of closure and/or a reflective statement

Informative and Explanatory Texts

a. include some sort of beginning to establish the topic (beyond using the title of the piece)
b. supply facts and information relevant to the topic
c. use simple additive linking words (e.g., *and*, *first*, *second*) to create connections between the facts
d. provide examples relevant to the topic
e. provide a sense of closure

Arguments (opinions)

a. introduce the topic or book directly, or use the title of the book as an introduction
b. state opinions (e.g., *My best friend is . . .*) relevant to the topic
c. provide reasons for opinions and details to support them

[9] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

d. use linking words that express causality (e.g., *I like . . . because . . .*)
e. refer to the content of the text when writing about literature

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively in group activities requiring speaking and listening.
2. Listen closely to and sustain attention on texts read aloud as well as other sources of information presented orally, visually, or multi-modally and confirm understanding by restating the information and answering pertinent questions.

Standards — Students can and do (by key communication type):

Classroom discussions and participating productively

a. initiate and participate in conversations with peers and adults through multiple exchanges, attending to the comments of others
b. confirm understanding by restating information or answering questions about what has been discussed
c. ask questions to get information, ask for help, or clarify something that is not understood
d. follow norms for conversation, such as listening to others and taking turns to speak

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Share experiences and ideas that demonstrate an awareness of their listeners.
4. Speak audibly and clearly.

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. describe people, places, things, and events with relevant facts and examples
b. recite or read aloud poems, rhymes, songs, and stories, speaking clearly at an understandable pace

Language Development Standards

Conventions

In grade 1, students gain increasing skill and independence in sentence formation and development. They have a sense of what a verb is and that its form changes to signal different time periods (past, present, and future). Their repertoire of prepositions continues to expand, and they use pronouns with regularity. Students capitalize names, places, and dates. They use end punctuation as well as commas in dates and in simple series of words. Their range of word-formation and spelling strategies grows.

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Key Terms: comma, pronoun, verb

Grammar and usage

Core Standards — Students can and do:

1. Produce and expand complete sentences in response to questions and prompts.
2. Use subject, object, and possessive pronouns (e.g., *I*, *me*, *my*; *they*, *them*, *their*).
3. Use verbs to convey a sense of past, present, and future in writing and speaking (e.g., *Today I walk home*; *Yesterday I walked home*; *Tomorrow I will walk home*).
4. Demonstrate understanding of frequently occurring prepositions (e.g., *during*, *beyond*, *toward*).

Mechanics

Core Standards — Students can and do:

5. Capitalize names, places, and dates.
6. Use end punctuation for sentences, including periods, question marks, and exclamation points.
7. Use commas in dates and to separate single words in a series.
8. Form new words through addition, deletion, and substitution of sound and letters (e.g., *an* → *man* → *mat* → *mast* → *must* → *rust* → *crust*).
9. Use conventional spelling for words with common spelling patterns and common irregular words.
10. Use phonetic spellings for untaught words, drawing on phonemic awareness and spelling conventions.

Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In grade 1, students begin to sort words themselves into categories rather than the objects that they name. They are able to define familiar words (e.g., *duck*) in a two-step process of identifying a category (bird) to which it belongs and naming one or more attributes that distinguish this category member from others (able to swim). Students grasp that many words they know can mean different things depending on how the word is used, and they make distinctions between and among closely related verbs and adjectives in terms of manner and intensity. They acquire new words through interactive language use, including informal talk, discussion, listening to and responding to texts read aloud as well as by being taught the words direct.

Determining the meaning of words

Core Standards — Students can and do:

1. Sort words into categories (e.g., colors, clothing).
2. Define words by category and by one or more key attributes (e.g., a *duck* is a bird that swims; a *tiger* is a large cat with stripes).
3. Demonstrate understanding of the concept of multiple-meaning words (e.g., *match*, *kind*, *play*) by identifying various meanings of some grade-appropriate examples of such words.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

4. Define, choose, or act out the meaning of closely related verbs that differ in manner (e.g., *look*, *peek*, *glance*, *stare*, *glare*, *scowl*; *speak*, *shout*, *mumble*, *whine*, *whimper*, *murmur*).
5. Distinguish among closely related adjectives that differ in intensity (e.g., *large*, *gigantic*; *hot*, *scalding*; *tasty*, *delicious*; *quiet*, *silent*).

Acquiring vocabulary

Core Standards — Students can and do:

6. Acquire and use new vocabulary taught directly and gained through conversations and hearing texts read aloud.

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English Language Arts

Grade 2

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Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grade

Grades	2–3 Level Text	4–5 Level Text
2	100%	
3	70%	30%

While advancing through the grades, students must engage with texts of steadily increasing complexity.

- **In grade 2**, students focus on reading texts in the 2–3 grade band level with scaffolding likely required for texts at the high end of the range.

Determining Text Complexity for Grades 2–3

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts[10]	Quantitative Measures of Texts
• *Structure:* Explicit, simple, conventional; simple graphic representations are supplementary to meaning; texts are relatively short • *Purpose:* Single; explicitly stated • *Style and Language:* Familiar, accessible, plain; few literary devices; mostly clear, everyday language; limited use of Tier 2 and 3 words and figurative language • *Richness:* A few ideas/concepts; concrete; low information density • *Relationships:* A few connections; explicit • *Knowledge Demands:* Ability to handle simple themes and fantastical elements as well as draw upon common, everyday experiences; general background knowledge and familiarity with genre conventions required; some everyday and general content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences as well as their interests, motivations, and maturity level.	

[10] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for Grade 2

Narratives	Drama	Poetry	Informational Text
At this level, includes children's adventure stories, biographies, folktales, legends, fables, fantasy, realistic fiction, and myth.	*At this level, includes staged dialogue, scenes, and brie, familiar scenes.*	*At this level, includes nursery rhymes and the subgenres of narrative poems, limericks, and free verse.*	*At this level, includes books about science, history, and the arts and other nonfiction materials.*

Illustrative Texts for Narratives, Drama, and Poetry[11]

Crow Boy by Taro Yashima (1955)

The Stories Julian Tells by Ann Cameron (1981)

Tops and Bottoms by Janet Stevens (1995)

"Grandpa's Stories" by Langston Hughes (1958)

"Weather" by Eve Merriam (1969)

Read alouds:

The Cricket in Times Square by George Selden, illustrated by Garth Williams (1960)

"Fireflies" by Paul Fleischman, illustrated by Eric Beddows (1988)

Illustrative Informational Texts

Maps & Globes by Jack Knowlton, pictures by Harriet Barton (1985)

Sunshine Makes the Seasons by Franklyn M. Branley (1985)

From Seed to Plant by Gail Gibbons (1991)

Boy, Were We Wrong About Dinosaurs by Kathleen V. Kudlinski, illustrated by S.D. Schindler (2005)

[11] See Appendix B for other texts illustrative of Grades 2-3 text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Retell what the text says explicitly and make inferences required to understand the text.
2. Identify the lessons or topics of the text and the key details that support them.
3. Describe in detail a specific character, event, or topic in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. ask and answer clarifying questions (e.g., how, why, where, when, who, and what) concerning specific details in the text and refer explicitly to parts of a text to answer these questions
b. identify or infer the moral or lesson in well-known stories, fables, folktales, or myths
c. describe how major events in a story often lead from problem to solution
d. examine a specific incident in a story, narrative, or drama in depth and establish when, where, and why it occurs
e. describe characters based upon what they say and do

Informational Text

a. accurately restate the key information provided by the text
b. ask and answer clarifying questions (e.g., how, why, and what) concerning specific details in the text and refer explicitly to parts of a text to answer these questions
c. identify the main idea and supporting details and facts in a text
d. explain the topic of each paragraph in a multi-paragraph text
e. identify specific events in historical or scientific texts and discuss what happened, as well as where, when, and why it happened, according to facts taken from the text

Reading Foundations

Phonics and Word Recognition

1. Students know and apply grade-level phonics and word analysis skills in decoding words.
   a. know the common spelling-sound correspondences for consonants (e.g., *wr-*, *sh*, *-ck*, *-ll*)
   b. know vowel digraph and final-*e* conventions for representing long vowels
   c. know spelling-sound correspondences for diphthongs and other common vowel teams (e.g., *loud*, *cow*, *look*, *loop*, *boy*, *boil*)
   d. use knowledge of phonics and spelling conventions to decode regularly spelled one-syllable words (e.g., *sick*, *march*, *sight*, *slice*, *bake*, *spring*)

Observing craft and structure

Core Standards — Students can and do:

4. Explain the meanings of words and phrases as they are used in the text.
5. Gain familiarity with different ways of presenting stories and information in text.
6. Compare and contrast different versions of the same story or informational texts on the same subject.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. recognize sensory details and how they are used to describe events, feelings, and objects
b. describe the different ways poets use rhyme, rhythm, and sensory images to convey a topic or message
c. identify repetitions in phrases, refrains, or sounds in poems and songs
d. describe story elements, including characters, setting, the problem, and how it is resolved
e. discuss stories written by the same author about similar characters or compare different versions of similar well-known tales and myths from various cultures

Informational Text

a. locate key words, facts, or other details using features of texts (e.g., captions, headings, glossaries, indexes, electronic menus, and icons)
b. distinguish between writing that is based on real events and writing that is based on fantasy or fictional events
c. combine information from two different parts of a text and identify how they are related (e.g., chronology, causation)
d. after reading two passages on the same subject, combine the information to more fully describe a topic

Reading Foundations, continued

Developing Fluency

2. Students read with sufficient accuracy and fluency to support comprehension.
   a. demonstrate increased accuracy, fluency, and expression on successive readings of a text
   b. use context to confirm or self-correct word recognition and understanding, rereading as necessary
   c. read alone or with a partner at least 20 minutes each day, in school or out

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Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Locate and use information from graphs, illustrations, and electronic sources.
8. Identify and understand words and phrases that indicate logical relationships.
9. Identify who is telling the story or providing information at any given point in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. efficiently navigate stories in print and electronic text and explain how images and illustrations connect to and clarify the content
b. identify who is telling the story or who is speaking in a drama

Informational Text

a. use information from visual elements of print and electronic texts (e.g., graphs, maps, charts, illustrations, photographs, diagrams) and explain how they help a reader understand the text
b. identify words (e.g., *such as*, *because*, *therefore*, *in order to*, *since*) that logically connect ideas in sentences and paragraphs

Developing habits for reading complex text

Core Standards — Students can and do:

10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding text.

Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and opinions that communicate to a familiar, known audience.

Conducting research

Core Standards — Students can and do:

2. Gather information from experiences or provided text sources.

Revising writing

Core Standards — Students can and do:

3. With specific guidance, strengthen writing through revision.

Using tools and technology

Core Standards — Students can and do:

4. Gain familiarity with technology and other tools to produce, revise, and edit writing.

Standards — Students can and do (by key text type):[12]

Narratives

a. establish a situation in time and/or place that is appropriate for the sequence of events to follow
b. recount a single well-elaborated event or sequence of events, managing chronological sequence with temporal words, phrases, and clauses
c. tell what the narrator thought or felt
d. include dialogue if appropriate and specific details
e. provide closure through reaction, commentary, or summation

Informative and Explanatory Texts

a. produce a brief introduction
b. create an organizational structure that presents similar information together, frequently patterned after chapter book headings or picture books
c. use adequate and specific facts and definitions to develop points
d. use linking words, such as *also*, *another*, *and*, and *more*, to connect ideas within categories of information, and headers to signal groupings
e. include a concluding sentence or section

Arguments (opinions)

a. introduce the topic or book(s) directly
b. state opinion(s) relative to the topic
c. provide reasons for opinions and details to support them
d. create a list-like structure for organization
e. use words to link and organize opinions and reason(s) (e.g., *because*, *another*, *and*, *also*)
f. refer to the text(s) when writing about literature
g. close with a concluding statement or recommendation

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively in small groups and as a class, engaging in a series of oral exchanges about texts and topics.

[12] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

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2. Sustain concentration on information presented orally, visually, or multi-modally and confirm understanding by paraphrasing the information.

Standards — Students can and do (by key communication type):

Classroom discussions and participating productively

a. engage in conversations on familiar topics
b. paraphrase the key information or ideas of others presented orally or through other media
c. inquire about oral or visual presentations to deepen understanding or clarify comprehension
d. link additions to conversation to the previous remarks of others
e. participate productively by listening politely to the ideas of others, taking turns speaking, and extending their ideas in light of discussions

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Share experiences and ideas, thinking about the needs of their listeners.
4. Speak audibly and clearly at an understandable pace.

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. recount stories or experiences with descriptive details by answering who, what, where, when, how, and why questions about them
b. report on a topic, including appropriate facts and details
c. use appropriate tone to express ideas, feelings, and needs clearly
d. recite or read aloud poems, rhymes, songs, and stories, speaking clearly at an understandable pace

Language Development Standards

Conventions

In grade 2, students create sentences of expanding length and complexity, though their control over these sentences is likely to be imperfect. Their command of noun and verb formation extends to common irregular forms. Students capitalize correctly in most situations. Their use of punctuation has grown to include commas in greetings and closings of letters as well as apostrophes to form contractions and to signal possession. Their spelling is increasingly conventional, and they now consult references, such as beginning dictionaries, to aid them when needed.

Key Terms: apostrophe, contraction, regular and irregular nouns and verb, possessive

Grammar and usage

Core Standards — Students can and do:

1. Generate and expand sentences with embedded, dependent, or conjoined clauses (e.g., *After we came home from school, I fed the gerbil and my sister cleaned the cage*).
2. Form common irregular plural nouns (e.g., *feet*, *children*, *teeth*, *mice*, *fish*, *women*).

3. Form the past tense of common irregular verbs (e.g., *sat*, *hid*, *told*).

Mechanics

Core Standards — Students can and do:

4. Capitalize holidays, product names, geographic names, and important words in titles.
5. Use commas in greetings and closings of letters.
6. Use apostrophes to punctuate contractions and to form common possessives.
7. Use conventional spelling for high-frequency and other studied words.
8. Generalize learned spelling patterns when writing words (e.g., *cage* → *badge*; *boy* → *boil*; *paper* → *copper*).
9. Use spelling rules for adding suffixes to base words (e.g., *sitting*, *smiled*, *cries*, *happiness*).
10. Consult reference materials, including beginning dictionaries, to check and correct spellings.

Vocabulary

In grade 2, students use a repertoire of strategies for dealing with unknown words. They can analyze the word itself, consider how it is used, consult reference materials, use the components of a compound word as clues to the word's meaning, or employ some combination of these strategies to determine or clarify word meanings. They figure out which meaning of a multiple-meaning word is most likely intended in a particular circumstance, and they differentiate among the connotations of related verbs and adjectives. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly.

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - identifying its base word when it has affixes (e.g., *happiness*, *finally*, *grimy*, *busily*)
   - determining how it is used in a sentence when reading, including whether it names or describes a thing or an action
   - consulting reference materials, including glossaries and beginning dictionaries, both print and digital
2. Determine the relevant meaning of multiple-meaning words by using context.
3. Explain the meaning of grade-appropriate compound words (e.g., *birdhouse*, *lighthouse*, *housefly*; *bookshelf*, *notebook*, *bookmark*).

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

4. Distinguish among related verbs (e.g., *toss*, *throw*, *hurl*) to gain a sense of their shadings of meaning.
5. Distinguish among related adjectives (e.g., *thin*, *slender*, *skinny*, *scrawny*; *irritated*, *mad*, *angry*, *furious*) to gain a sense of their shadings of meaning.

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Acquiring vocabulary

Core Standards — Students can and do:

6. Acquire and use new vocabulary taught directly and gained through reading and conversations.

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English Language Arts

Grade 3

Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grade

Grades	2–3 Level Text	4–5 Level Text
2	100%	
3	70%	30%

While advancing through the grades, students must engage with texts of steadily increasing complexity.

- **In grade 3**, students focus on reading texts in the 2–3 grade band level text (70 percent) independently and are introduced to texts in the 4–5 grade band level as "stretch" texts (30 percent), which will likely require scaffolding.

Determining Text Complexity for Grades 2–3

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts[13]	Quantitative Measures of Texts
• *Structure:* Explicit, simple, conventional; simple graphic representations are supplementary to meaning; texts are relatively short • *Purpose:* Single; explicitly stated • *Style and Language:* Familiar, accessible, plain; few literary devices; mostly clear, everyday language; limited use of Tier 2 and 3 words and figurative language • *Richness:* A few ideas/concepts; concrete; low information density • *Relationships:* A few connections; explicit • *Knowledge Demands:* Ability to handle simple themes and fantastical elements as well as draw upon common, everyday experiences; general background knowledge and familiarity with genre conventions required; some everyday and general content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences as well as their interests, motivations, and maturity level.	

[13] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for Grade 3

Narratives	Drama	Poetry	Informational Text
At this level, includes children's adventure stories, biographies, folktales, legends, fables, fantasy, realistic fiction, and myth.	*At this level, includes staged dialogue, scenes, and brief familiar scenes.*	*At this level, includes nursery rhymes, and the subgenres of narrative poems, limericks, and free verse.*	*At this level, includes books about science, history, and the arts and other nonfiction materials.*

Illustrative Texts for Narratives, Drama, and Poetry[14]

My Father's Dragon by Ruth Stiles Gannett, illustrated by Ruth Chrisman Gannett (1948)

Sarah, Plain and Tall by Patricia MacLachlan (1985)

The One-Eyed Giant (Book One of Tales from the Odyssey) by Mary Pope Osborne (2002)

"Knoxville, Tennessee" by Nikki Giovanni (1968)

"Eating While Reading" by Gary Soto (1995)

Read Alouds:

"How the Camel Got His Hump" in *Just So Stories* by Rudyard Kipling (1902)

Illustrative Informational Text

A Medieval Feast by Aliki (1983)

So You Want to Be President? by Judith St.George, illustrated by David Small (2000)

Bat Loves the Night by Nicola Davies, illustrated by Sarah Fox-Davies (2008)

Moonshot: The Flight of Apollo 11 by Brian Floca (2009)

[14] See Appendix B for other texts illustrative of Grades 2-3 text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Retell what the text says explicitly and make inferences required to understand the text.
2. Identify lessons or topics of the text and the key details that support them.
3. Describe in detail a specific character, event, or topic in the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. ask and answer clarifying questions (e.g., how, why, where, when, who, and what) concerning specific details in the text and refer explicitly to parts of a text to answer these questions
b. identify or infer the moral or lesson in well-known stories, fables, folktales, or myths
c. describe how major events in a story often lead from problem to solution
d. examine a specific incident in a story, narrative, or drama in depth, and establish when, where, and why it occurs
e. describe characters based upon what they say and do

Informational Text

a. accurately restate key information provided by the text
b. ask and answer clarifying questions (e.g., how, why, and what) concerning specific details in the text and refer explicitly to parts of a text to answer these questions
c. identify the main idea and supporting details and facts in a text
d. explain the topic of each paragraph in a multi-paragraph text
e. identify specific events in historical or scientific texts and discuss what happened, as well as where, when, and why it happened, according to facts taken from the text

Reading Foundations

Phonics and Word Recognition

1. Students know and apply grade-level phonics and word analysis skills in decoding words.
   a. identify and know the meaning of the most common prefixes and derivational suffixes (e.g., *un-*, *re-*, *mis-*, *-ful*, *-tion*, *-able)*
   b. decode regularly spelled single-syllable and multi-syllable words (e.g., *vocabulary*, *refrigerator*, *terrible*, *frightening*)
   c. read grade-appropriate irregularly spelled words by sight
   d. use phonics and word analysis to identify visually new words when reading

Observing craft and structure

Core Standards — Students can and do:

4. Explain the meanings of words and phrases as they are used in the text.
5. Gain familiarity with different ways of presenting stories and information in text.
6. Compare and contrast different versions of the same story or informational texts on the same subject.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. recognize sensory details and how they are used to describe events, feelings, and objects
b. describe the different ways poets use rhyme, rhythm, and sensory images to convey a topic or message
c. identify repetitions in phrases, refrains, or sounds in poems and songs
d. describe story elements, including characters, setting, the problem, and how it is resolved
e. discuss stories written by the same author about similar characters or compare different versions of similar well-known tales and myths from various cultures

Informational Text

a. locate key words, facts, or other details using features of texts (e.g., captions, headings, glossaries, indexes, electronic menus, and icons)
b. distinguish between writing that is based on real events and writing that is based on fantasy or fictional events
c. combine information from two different parts of a text and identify how they are related (e.g., chronology, causation)
d. after reading two passages on the same subject, combine the information to more fully describe a topic

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Locate and use information from graphs, illustrations, and electronic sources.
8. Identify and understand words and phrases that indicate logical relationships.
9. Identify who is telling the story or providing information at any given point in the text.

Reading Foundations, continued

Developing Fluency

2. Students read with sufficient accuracy and fluency to support comprehension.
   a. demonstrate increased accuracy, fluency, and expression on successive readings of a text
   b. use context to confirm or self-correct word recognition and understanding, rereading as necessary
   c. read at least 20 minutes each day, in school or out

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Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. efficiently navigate stories in print and electronic text and explain how images and illustrations connect to and clarify the content
b. identify who is telling the story or who is speaking in a drama

Informational Text

a. use information from visual elements of print and electronic texts (e.g., graphs, maps, charts, illustrations, photographs, diagrams) and explain how they help a reader understand the text
b. identify words (e.g., *such as*, *because*, *therefore*, *in order to*, *since*) that logically connect ideas in sentences and paragraphs

Developing habits for reading complex text

Core Standards — Students can and do:

10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding text.

Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and opinions that communicate to a familiar, known audience.

Conducting research

Core Standards — Students can and do:

2. Gather information from experiences or provided text sources.

Revising writing

Core Standards — Students can and do:

3. With specific guidance, strengthen writing through revision.

Using tools and technology

Core Standards — Students can and do:

4. Gain familiarity with technology and other tools to produce, revise, and edit writing.

Standards — Students can and do (by key text type):[15]

Narratives

a. set the time, indicate a location, introduce characters, or enter immediately into the story line to engage the reader
b. recount a single, well-elaborated event or a sequence of events that unfold naturally using temporal words, phrases, and clauses
c. tell what the narrator thought or felt
d. develop a focus, provide pacing, and include only relevant information
e. develop a character through the description of external behavior
f. provide descriptive details
g. employ dialogue and other narrative strategies
h. provide a satisfying conclusion that is reflective and/or that effectively ties up loose ends

Informative and Explanatory Texts

a. produce an introduction that names the topic and provides at least one general detail about it
b. create an organizational structure that presents similar information together, frequently patterned after chapter book headings or picture books
c. use adequate, relevant, and specific facts and definitions to develop points
d. logically categorize details and facts drawn from personal experience and other sources
e. use linking words, such as *also*, *another*, *and*, and *more*, to connect ideas within categories of information, and use headers to signal groupings
f. include only appropriate information
g. include a concluding sentence or section

Arguments (opinions)

a. introduce the topic or book(s) directly, and attempt to capture the reader's interest
b. state an opinion relative to the topic (e.g., *This is a good book* or *John is a good friend*)
c. provide facts and details to support the opinion
d. create a list-like organizing structure that provides reasons for the opinion
e. use appropriate words to link and organize opinions and reason(s) (e.g., *because*, *another*, *and*, *also*)
f. refer to the text(s) when writing about literature
g. provide a concluding statement, reflection, and/or recommendation

[15] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

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Speaking and Listening Standards

Listening critically and participating productively

Core Standards — Students can and do:

1. Participate productively in small groups and as a class, engaging in a series of oral exchanges about texts and topics.
2. Sustain concentration on information presented orally, visually, or multi-modally and confirm understanding by paraphrasing the information.

Standards — Students can and do (by key communication type):

Classroom discussions and participating productively

a. engage in conversations on familiar topics
b. paraphrase the key information or ideas of others presented orally or through other media
c. inquire about oral or visual presentations to deepen understanding or clarify comprehension
d. link additions to conversation to the previous remarks of others
e. participate productively by listening politely to the ideas of others, taking turns speaking, and extending their ideas in light of discussions

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Share experiences and ideas, thinking about the needs of their listeners.
4. Speak audibly and clearly at an understandable pace.

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. recount stories or experiences with descriptive details by answering who, what, where, when, how, and why questions about them
b. report on a topic, including appropriate facts and details
c. use appropriate tone to express ideas, feelings, and needs clearly
d. recite or read aloud poems, rhymes, songs, and stories, speaking clearly at an understandable pace

Language Development Standards

Conventions

By grade 3, students have learned the foundations of written and spoken language, including letter, word, and sentence formation and crucial forms of punctuation. They ensure agreement between subject and verb and between pronoun and antecedent in simple situations. Students use quotation marks to indicate dialogue. They know most of the conventions of spelling and consult references to look up words when they still have difficulty. They use precise everyday language to describe and begin to consider the effects of word choice in writing and speaking.

Key Terms: subject-verb and pronoun-antecedent agreement, comma splice, fragment, run-on, quotation mark

Conventions of language and writing

Core Standards — Students can and do:

1. Group related ideas into a paragraph.

Grammar and usage

Core Standards — Students can and do:

2. Generate complete sentences, avoiding fragments, comma splices, and run-ons.*
3. Ensure subject-verb and pronoun-antecedent agreement.*

Mechanics

Core Standards — Students can and do:

4. Use quotation marks in dialogue.
5. Use spelling patterns and generalizations (e.g., word families, position-based spellings, syllable patterns, ending rules, meaningful word parts) in writing regular words.
6. Consult reference materials, including dictionaries, to check and correct spellings.

Word choice and style

Core Standards — Students can and do:

7. Use precise everyday language.
8. Choose words for effect.*[16]

[16] Conventions standards noted with an asterisk (*) need to be revisited by students in subsequent grades. See Appendix A for a full listing.

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Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In grade 3, students use their repertoire of strategies to determine and clarify the meaning of unknown and multiple-meaning words. They know that words are sometimes used in nonliteral ways and can use that knowledge to help them understand common idioms. They learn and can paraphrase many common idioms and sayings. They recognize that words have nuances in meaning and rely on context and background knowledge to sort among related words that describe abstract concepts. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly.

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - using prefixes and suffixes when it is a multimorpheme word (e.g., *thoughtless*, *recycle*, *unforgettable*)
   - determining how it is used in a sentence when reading
   - consulting reference materials, including glossaries and dictionaries, both print and digital
2. Determine the meaning of multiple-meaning words by using context.
3. Distinguish between literal and nonliteral uses of language.
4. Paraphrase the meaning of common idioms and sayings.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

5. Distinguish among related words that describe states of mind, degrees of certainty, or other abstract concepts (e.g., *knew*, *believed*, *suspected*, *heard*, *wondered*).

Acquiring vocabulary

Core Standards — Students can and do:

6. Acquire and use new vocabulary taught directly and gained through reading and conversations.

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English Language Arts

Grades 4–5

Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grade

Grades	4–5 Level Text	6–8 Level Text
4	100%	
5	70%	30%

While advancing through grades 4–5, students must engage with texts of steadily increasing complexity.

- **In grade 4**, students focus on reading texts in the 4–5 grade band level with scaffolding likely required for texts at the high end of the range.
- **In grade 5**, students focus on reading in the 4–5 grade band level (70 percent) independently and are introduced to texts in the 6–8 grade band level as "stretch" texts (30 percent), which will likely require scaffolding.

Determining Text Complexity for Grades 4–5[17]

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts	Quantitative Measures of Texts
• *Structure:* Largely explicit and direct; graphic representations are supplementary to meaning; texts are of increasing length • *Purpose:* Single or twofold; clearly indicated • *Style and Language:* Moderately accessible; some literary devices; some everyday language; some use of Tier 2 and Tier 3 words and figurative language • *Richness:* Some ideas/concepts; mostly concrete; moderate information density • *Relationships:* Some connections; largely explicit • *Knowledge Demands:* Ability to handle fairly simple themes, consider a perspective somewhat different from one's own, and understand unfamiliar experiences; general background knowledge and familiarity with genre conventions required; some general and discipline-specific content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences as well as their interests, motivations, and maturity level.	

[17] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for Grades 4–5

Narratives	Drama	Poetry	Informational Text
At this level, includes children's adventure stories, biographies, folktales, legends, fables, fantasy, realistic fiction, and myth.	*At this level, includes staged dialogue and brief familiar scenes.*	*At this level, includes nursery rhymes, and the subgenres of narrative poems, limericks, and free verse.*	*At this level, includes books about science, history, and the arts and other nonfiction materials and digital media sources on a range of topics.*

Illustrative Texts for Narratives, Drama, and Poetry[18]	Illustrative Informational Texts
Alice in Wonderland by Lewis Carroll (1865) *The Little Prince* by Antoine de Saint-Exupery (1943) *Bud, Not Buddy* by Christopher Paul Curtis (1999) "The Echoing Green" from *Songs of Innocence* by William Blake (1789) "Casey at the Bat" by Ernest Lawrence Thayer (1888) "Words Free As Confetti" by Pat Mora (1996) *Where the Mountain Meets the Moon* by Grace Lin (2009)	*Discovering Mars* by Melvin Berger (1992) *Hurricanes: Earth's Mightiest Storms* by Patricia Lauber (1996) "Ancient Mound Builders" by E. Barrie Kavash from *Cobblestone* (2003) *Volcanoes* by Seymour Simon (2006) "Kenya's Long Dry Season" by Nellie Gonzalez Cutler from *Time for Kids* (2009) "Seeing Eye to Eye" by Leslie Hall from *National Geographic Explorer* (2009)

[18] See Appendix B for other texts illustrative of Grades 4-5 text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Determine what the text says explicitly and make inferences required for understanding; explain how those inferences stem from the text.
2. Articulate the main ideas and themes of the text and provide a summary that captures the key supporting details.
3. Describe in detail two or more characters, events, or topics in the text and explain how they are related to one another.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. determine the theme of a story or drama, basing the understanding of theme on how characters adapt or change in response to the challenges posed in the plot
b. summarize accurately the significant events of a play or narrative in chronological order, describing where, when, why, and how specific actions take place
c. describe characters based on evidence from their thoughts, words, deeds, and interactions with others
d. describe the setting in detail, drawing on evidence of the time, place, and other cues
e. determine the theme of a poem, basing the understanding of theme on the key observations, images, or statements in a poem

Informational Text

a. outline the main and supporting ideas in the text and provide an accurate summary
b. identify the topic sentence and gist of each paragraph in a multi-paragraph text
c. describe related events in a history text or related topics in a science text and explain the relationships between the events or topics

Observing craft and structure

Core Standards — Students can and do:

4. Explain the meanings of words and phrases in the text, distinguishing literal and figurative uses.
5. Comprehend literature and information presented in a range of structures.
6. Compare and contrast texts written on the same topic or theme and explain how they are different and similar.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. describe the sensory details in texts and distinguish the use of literal versus figurative language
b. observe and explain how words with similar meanings can have different connotations
c. identify the meaning of figurative phrases and culturally significant characters found in mythology that are integral to understanding other works of literature and texts (e.g., *Herculean*, *Pandora's box*)
d. identify how narratives and plays are structured to describe the progress of characters through a series of events and challenges

e. identify rhymes and other repetitions of sounds that supply rhythm and pattern in poems and narrative prose
f. compare a narrative or a play with a presentation in another format, such as film, stage, or interactive text, and note what is surprising or different about the alternative version
g. compare works of literature on the same topic or with a similar theme

Informational Text

a. explain the meaning of key words and terms as they are used in the text
b. understand information drawn from a variety of texts with different structures, such as chronological, compare-and-contrast, or as a chain of causes and effects
c. identify and use text features (e.g., bold print, key words, topic sentences, hyperlinks, electronic menus, and icons) to locate information quickly and aid in comprehension
d. compare and contrast related accounts on the same or similar topics by different authors, by analyzing their content and perspectives

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Explain and use information presented graphically or visually in print, videos, or electronic texts.
8. Outline the information or evidence used to support an explanation or argument, determining which points support which key statements.
9. Determine the point of view or purpose that guides how events or ideas are described.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. identify the narrator of a story and explain how different stories are narrated from different perspectives
b. compare accounts of historical events and figures or natural phenomena with their depiction in a fictional work

Informational Text

a. explain how factual information presented graphically or visually (e.g., maps, charts, diagrams, timelines, animations, and other interactive visual elements) aids in the comprehension of print and electronic texts
b. explain how authors support their specific claims with evidence, including which evidence supports which claims
c. determine the author's purpose and how that is reflected in the description of the events and ideas

Developing habits for reading complex text

Core Standards — Students can and do:

10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding texts.

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Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and arguments that demonstrate an awareness of audiences that are familiar and known to the student.

Conducting research

Core Standards — Students can and do:

2. Perform short, focused research tasks that build knowledge by exploring aspects of a single topic.
3. Gather information from experience, as well as print and digital resources.
4. Determine the accuracy and relevance of the information gathered to answer specific questions.
5. Restate information from source materials in one's own words, through summary or paraphrase.
6. Provide basic bibliographic information for print and digital sources.

Revising writing

Core Standards — Students can and do:

7. With guidance and support from peers and adults, strengthen writing through revision, editing, or beginning again to maintain a clear focus throughout.

Using tools and technology

Core Standards — Students can and do:

8. Use technology and other tools to produce, revise, and edit writing.

Developing proficiency in a range of writing

9. Create writing over extended timeframes (time for reflection and revision) and shorter timeframes (a single sitting or a day or two), responding to specific sources.

   Focus by grade level:

   Grade 4: Describing the content of literary or informational sources at the 4-5 grade band level of text complexity and content

   Grade 5: Comparing the contents of literary or informational sources at the 4-5 grade band level of complexity and content

Standards — Students can and do (by key text type):[19]

Narratives

a. orient the reader by establishing a situation, introducing characters, setting, and location, or by backfilling information after entering immediately into the storyline
b. create an organizing structure in which events are logically or causally sequenced
c. in producing a story, create a plot with an initiating event, complicating action, a climax, and a resolution
d. use a variety of temporal words, phrases, and clauses to signal sequence
e. use concrete and sensory details to develop narrative elements
f. develop the narrative using techniques such as dialogue, pacing, and reporting the narrator's thoughts
g. show both external behaviors and the internal responses of characters to events
h. provide closure and a realistic outcome of the narrative's events

Informative and Explanatory Texts

a. state the topic clearly and provide a general observation and focus
b. develop the subject using relevant facts, concrete details, quotations, or other information and examples
c. group related information logically in basic structures (paragraphs, sections) and provide headings or illustrations when useful
d. employ specialized vocabulary and a formal, objective style when appropriate
e. use appropriate links to join ideas
f. include only relevant appropriate information to demonstrate focus
g. provide a conclusion related to the information or explanation offered

Arguments (opinions)

a. introduce an opinion about a concrete issue or topic
b. support opinions with relevant reasons
c. support reasons with specific details
d. link the reasons together using words, phrases, and clauses (e.g., *because*, *since*)
e. adopt a relatively formal style for sharing and defending an opinion when appropriate to the discipline or context
f. provide a concluding statement or section that offers reflections, restatement, or recommendations consistent with the opinion presented

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively one on one, in small groups, and as a whole class, joining in discussions and making relevant points about what they have read, heard, or written.
2. Sustain concentration on information presented orally, visually, or multi-modally and confirm understanding by summarizing the main ideas and supporting details.

Standards — Students can and do (by key communication type):

[19] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

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Classroom discussions and collaboration

a. come to discussions having read required material and, in conversation, build upon background knowledge from that material and other information known about the topic
b. demonstrate understanding of the content and ideas presented or discussed by distilling them into an accurate summary
c. ask questions to clarify or follow up on ideas or information presented orally or through other media
d. respond to questions and make comments that contribute to the topic and ideas of previous speakers
e. explain information presented graphically or visually in conjunction with other information presented orally
f. engage productively and respectfully with others during discussions, including listening actively, gaining the floor respectfully, and qualifying or justifying what they think after listening to others' questions or accounts

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Share experiences, opinions or other information, choosing material that is relevant to the topic and to the listeners.
4. Speak audibly and clearly at an appropriate and understandable pace, using formal English when indicated or appropriate (e.g., presenting ideas versus class discussion).

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. speak coherently about events, topics, or texts that focus and organize ideas in a logical sequence and include facts, details, or other information that support the main ideas
b. use appropriate volume, phrasing, and pace for clarity
c. read aloud prose and poetry, with appropriate emotion and fidelity to the text

Language Development Standards

Conventions

In grades 4–5, students heighten their ability to situate and describe using language that is increasingly precise and vivid. They form and use verbs of various tenses to locate people, actions, and events in time, and they correctly use adjectives and adverbs to modify. Students begin to gain control of frequently confused words (e.g., *effect*, *affect*) and edit writing to remove language that is not idiomatic. Their mastery of capitalization is complete. They use punctuation to separate items in a series and a comma to distinguish an introductory element from the main part of the sentence. Students mark titles in conventional ways. They understand how to quote and use quotation marks. Their spelling is conventional. Their language is increasingly topic specific, precise, and varied, and they manipulate sentence structure for effect.

Key Terms: adjective; adverb; interjection; preposition; simple, progressive, and perfect tense

Conventions of language and writing

Core Standards — Students can and do:

1. Maintain the focus of a paragraph on a topic through structural elements such as main ideas, supporting sentences, and transitions.

Grammar and usage

Core Standards — Students can and do:

2. Form and use the simple (e.g., *I walked*, *I walk*, *I will walk*), progressive (e.g., *I was walking*, *I am walking*, *I will be walking*) and the perfect (e.g., *I had walked*, *I have walked*, *I will have walked*) verb tenses.
3. Recognize and correct inappropriate shifts in verb tense.*
4. Form and choose between adjectives and adverbs (including comparative and superlative forms), placing them appropriately within the sentence.*
5. Correctly use frequently confused words.*
6. Use idiomatic language.*

Mechanics

Core Standards — Students can and do:

7. Capitalize the first word in quotations as appropriate and other important words, such as section headers.
8. Use punctuation to separate items in a series.*
9. Use a comma to separate an introductory element from the rest of the sentence.
10. Use underlining, quotation marks, or italics to indicate titles of works.
11. Use quotation marks to mark direct speech and quotations from a text.
12. Spell grade-appropriate words correctly, consulting references as needed.*

Word choice and style

Core Standards — Students can and do:

13. Use specialized, topic-specific language to convey ideas precisely.*
14. Use figurative language to create images or make comparisons and connections between people, objects, or ideas.*
15. Use punctuation for effect.*
16. Expand, combine, and reduce sentences for meaning, reader/listener interest, and style.*[20]

Focus by Grade-Level
Grade 4: Distinguish one idea or thing from another (Conventions Standards #'s1-3, #8, #9, #11)
Grade 5: Word choice (Conventions Standards #'s 4-6, #13, #14)

[20] Conventions standards noted with an asterisk (*) need to be revisited by students in subsequent grades. See Appendix A for a complete listing.

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Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In grades 4–5, students are capable of selecting among a wide range of strategies—analyzing the word itself, using localized context clues (particularly at the sentence level), and consulting reference materials—to determine and clarify the meaning of unknown and multiple-meaning words. They develop the habit of verifying their inferences of word meanings. They are able to interpret simple figurative language found in what they read. They learn and can paraphrase many common idioms, proverbs, and adages. They make distinctions among words based on connotation. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly. This includes a focus on "Tier 2" words and phrases (those that commonly appear in writing but not in spoken language), "Tier 3" words and phrases (those that are specific and important to particular disciplines).[21]

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - analyzing the word's sounds, spelling, and meaningful word parts
   - using semantic clues in sentences, such as definitions, examples, or restatements included within the text
   - using syntactic clues, such as using its position within the sentence as a guide to whether it represents a thing or an action
   - consulting reference materials, including glossaries, dictionaries, and thesauruses, both print and digital
2. Determine the relevant meaning of multiple-meaning words by using context.
3. Verify the preliminary determination of a word's meaning (e.g., by checking the inferred meaning in context or by looking up the word in a dictionary).
4. Interpret figurative language, including simple similes and metaphors.
5. Paraphrase the meaning of common idioms, adages, and proverbs.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

6. Distinguish a word from other words with similar but not identical meanings (synonyms).

Acquiring vocabulary

Core Standards — Students can and do:

7. Acquire and use a grade-appropriate vocabulary of Tier 2[22] words taught directly and gained through reading.
8. Acquire and use a grade-appropriate vocabulary of Tier 3 words taught directly and gained through reading.
9. Know and use words and phrases that signal contrast, addition, or other logical relationships (e.g., *however*, *although*, *nevertheless*, *similarly*, *moreover*, *in addition*).

[21] Beck I. L., McKeown, M.G. & Kucan, L. (2002). *Brining Words to Life; Robust Vocabulary Instruction*, New York; Guilford Press

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Grade 4 English Language Arts: Focus for Instruction

Reading and Literature

In grade 4, students apply the reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read text at the 4-5 grade band level independently, with scaffolding likely required for texts at the high end of the range.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 100% text at the 4-5 grade band level

100%

Writing and Research

In grade 4, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by describing the contents of literary or informational sources at the 4-5 grade band level of complexity and content

Speaking and Listening

In grade 4, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentation of ideas and information

Language Development

In grade 4, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Distinguish one idea or thing from another:
  - Maintain the focus of a paragraph on a topic… (Conventions Standard #1)
  - Form and use the simple, progressive and perfect verb tenses … (Conventions Standard #2)
  - Recognize and correct inappropriate shifts in verb tense. (Conventions Standard #3)
  - Use punctuation to separate items in a series … (Conventions Standard #8)
  - Use a comma to separate an introductory element … (Conventions Standard #9)
  - Use quotation marks to mark direct speech and quotations … (Conventions Standard #11)

Grade 5 English Language Arts: Focus for Instruction

Reading and Literature

In grade 5, students apply the reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read text at the 4-5 grade band level independently and are introduced to 6-8 grade band "stretch" texts, which will likely require scaffolding.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 70% text at the 4-5 grade band level, 30% text at the 6-8 grade band level

Writing and Research

In grade 5, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. There also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by comparing the contents of literary or informational sources at the 4-5 grade band level of complexity and content.

Speaking and Listening

In grade 5, students apply the speaking and listening standards in different contexts.

- Speaking and listening Standards applied in different contexts: classroom discussion and collaboration as well as in presentation of ideas and information.

Language Development

In grade 5, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions,

- Vocabulary standards applied to both reading, writing, speaking and listening
- **Grade-specific conventions focus:** Word choice
  - Form and choose between adjectives and adverbs. (Conventions Standard #4)
  - Correctly use frequently confused words. (Conventions Standard #5)
  - Use idiomatic language. (Conventions Standard #6)
  - Use specialized, topic specific language to convey ideas precisely. (Conventions Standard #13)
  - Use figurative language to create images … (Conventions Standard #14)

English Language Arts

Grades 6–8

Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grade

While advancing through grades 6–8, students must engage with texts of steadily increasing complexity.

- **In grade 6**, students focus on reading texts in the 6–8 grade band level with scaffolding likely required for texts at the high end of the range.
- **In grade 7**, students focus on reading texts in the 6–8 grade band level (90 percent) independently and are introduced to texts in the 9–10 grade band level as "stretch" texts (10 percent), which will likely require scaffolding.
- **In grade 8**, students focus on reading texts in the 6–8 grade band level (70 percent) independently as well as sustained practice with texts in the 9–10 grade band level as "stretch" texts (30 percent), which will likely require scaffolding.

Determining Text Complexity for Grades 6–8[23]

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts	Quantitative Measures of Texts
• *Structure:* Largely implicit and subtle; graphic representations are essential to meaning; texts are of increasing length • *Purpose:* Single or multiple; subtly stated • *Style and Language:* Moderately demanding; several literary devices; consistent use of Tier 2 and 3 words and figurative language • *Richness:* Several ideas/concepts; mostly abstract; moderate information density • *Relationships:* Several connections; largely implicit • *Knowledge Demands:* Ability to handle fairly challenging themes, consider multiple perspectives, and understand unfamiliar experiences; cultural and historical knowledge useful for understanding characters, settings, and allusions; some discipline-specific content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences as well as their interests, motivations, and maturity level.	

[23] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for Grades 6–8

Narratives	Drama	Poetry	Informational Text
At this level, includes the subgenres of adventure stories, biographies, memoirs, historical fiction, mysteries, folktales, legends, fables, tall tales, myths, fantasy, science fiction, realistic fiction, and graphic novels.	*At this level, includes one-act and multi-act plays both as text and film.*	*At this level, includes the subgenres of narrative poems, lyrical poems, free verse, odes, ballads, and epics.*	*At this level, includes such subgenres as exposition and argument in the form of essays, opinion pieces, speeches, opinion pieces as well as other documents and digital media sources on a range of topics.*

Illustrative Texts for Narratives, Drama, and Poetry[24]	Illustrative Informational Texts
Little Women by Louisa May Alcott (1869) *The Adventures of Tom Sawyer* by Mark Twain (1876) *A Wrinkle in Time* by Madeline L'Engle (1962) *The Dark Is Rising* by Susan Cooper (1973) *Black Ships before Troy: The Story of the Iliad* by Rosemary Sutcliff (1993) *A Midsummer Night's Dream* by William Shakespeare (1596) "Oh Captain, My Captain" by Walt Whitman (1865) "Stopping by a Wood on a Snowy Evening" by Robert Frost (1923) "I, Too" by Langston Hughes (1925)	*Preamble and First Amendment to the United States Constitution* by United States (1787, 1791) ** *Narrative of the Life of Frederick Douglass an American Slave* by Frederick Douglass (1845) "Gettysburg Address"** by Abraham Lincoln (1863) "Blood, Toil, Tears and Sweat" by Winston Churchill (1940) *Travels with Charley: In Search of America* by John Steinbeck (1962) *I Know Why the Caged Bird Sings* by Maya Angelou (1969)

*****Seminal historical texts that all students are expected to read***

[24] See Appendix B for other texts illustrative of Grades 6-8 text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Read the text closely to determine what the text says explicitly and to make logical inferences from it; cite text evidence to support understanding in discussion and in writing.
2. Articulate the text's main ideas and themes and provide a summary that captures the key supporting details, without taking a position or expressing an opinion.
3. Explain in detail how events, ideas, and characters unfold in the text and interact with one another.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. infer themes when they are not explicitly stated and provide evidence on which those inferences are based
b. analyze the development of the narrative, describing how particular incidents advance or foreshadow the plot
c. recognize how the setting unfolds over the course of the text and describe its significance to the work
d. build on an author's explicit descriptions and other evidence to draw reasonable conclusions about characters and how they interact, change, and influence the central events
e. describe how a play unfolds and how particular lines of dialogue propel the action, reveal aspects of a character, or provoke a decision
f. analyze how patterns of imagery in a poem contribute to its overall theme or meaning

Informational Text

a. summarize a text without expressing a personal opinion by drawing on the author's specific description of events or information
b. determine how key ideas or concepts build on one another to reveal an overarching theme or idea

Observing craft and structure

Core Standards — Students can and do:

4. Interpret the meanings of words and phrases, including connotative and figurative meanings, and explain how specific word choices shape the meaning of the text.
5. Explain the text's structure, including how specific sentences, paragraphs, and larger portions build on each other and contribute to the whole of the text.
6. Compare and contrast how two or more texts written on similar topics or themes differ in their focus and key details.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. analyze how the author's choice of specific words or details contributes to the understanding of events and characters or to the tone of a narrative
b. trace the specific comparisons made by similes, metaphors, and analogies and explain how they contribute to the meaning of the text
c. compare similar ideas and themes as well as character types in myths, folktales, and legends from different cultures

d. analyze the impact of line breaks and stanzas on the meaning of a poem and acts, scenes, and stage directions on the meaning of a drama
e. compare the events, characters, ideas, and themes in texts written by the same author or on similar topics or themes

Informational Text

a. interpret the connotative meaning of closely related words and phrases as they are used in the text (e.g., *angry* versus *irate*)
b. describe how an author organizes the explanation or argument, as well as the ways in which the text's structure, language, and examples support its purpose
c. examine the structure of a Web site or other electronic text and describe how it organizes information and links to additional sources

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Interpret information presented graphically or visually in print, videos, or electronic texts and explain how this information clarifies and contributes to the text.
8. Analyze the structure and content of an argument, including its main claims or conclusions, supporting premises, and evidence.
9. Determine the point of view or purpose represented in the text, assessing how it shapes the content.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. compare the points of view from which different novels and poems are told, as well as the viewpoints of different characters in a drama
b. compare the fictional portrayal of a time, place, or character to historical sources from the period to determine which historical details have been emphasized, deleted, or changed in the fictional portrayal

Informational Text

a. interpret factual and quantitative data presented in diverse formats (including maps, charts, and diagrams as well as electronic media) and explain how this information clarifies or contributes to the text
b. distinguish between fact, opinion, and reasoned judgment presented in essays, speeches, and critiques
c. evaluate the strength of an argument's premises and specific claims as well as the degree to which each is supported by evidence
d. compare and contrast the viewpoints and use of evidence of two different authors writing about the same topic

Developing habits for reading complex text

Core Standards — Students can and do:

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10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding texts.

Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write narratives, informative and explanatory texts, and arguments that match purpose to task and address familiar as well as somewhat distant audiences (e.g., mayor, readers of school or neighborhood newspaper).

Conducting research

Core Standards — Students can and do:

2. Perform short, focused research projects that demonstrate understanding of the material under investigation and generate additional related questions for research.
3. Gather information independently using a variety of relevant print and digital resources.
4. Assess the credibility, reliability, consistency, and accuracy of the information and sources gathered.
5. Represent and cite accurately the data, conclusions, and opinions of others, quoting and paraphrasing them into one's own work while avoiding plagiarism.
6. Provide full bibliographic information for print and digital sources in a standard format and document quotations, paraphrases, and other information.

Revising writing

Core Standards — Students can and do:

7. With some guidance and support from peers and adults, strengthen writing through revising, editing, or beginning again to ensure logical organization, precision of word choice, and coherence.

Using tools and technology

Core Standards — Students can and do:

8. Use technology and other tools to produce, revise, and distribute writing, as well as interact online with others about writing, including responding to and providing feedback

Developing proficiency in a range of writing

9. Create writing over extended timeframes (time for reflection and revision) and shorter timeframes (a single sitting or a day or two), responding to specific sources.

   Focus by grade level:

   Grade 6: Conveying the main ideas and key details of literary or informational sources at the 6-8 grade band level of text complexity and content
   Grade 7: Analyzing the contents of literary or informational sources at the 6-8 grade band level of complexity and content
   Grade 8: Comparing or evaluating the contents of literary or informational sources at the 6-8 grade band level of complexity and content

Standards — Students can and do (by key text type):[25]

Narratives

a. orient the reader by establishing a situation, introducing characters, setting, and location, or by backfilling information after entering immediately into the storyline
b. create an organizing structure in which events are logically or causally sequenced
c. in producing a story, create a plot with well-structured episodes (e.g., initiating event, complicating action, resolution)
d. use a variety of temporal words, phrases, and clauses to convey sequence, to shift from one time frame to another, and to show the relationships among events
e. use relevant, specific details and literary devices, such as imagery and metaphor, purposefully to develop setting, plot, and character
f. use techniques such as pacing, dialogue, or foreshadowing to highlight the significance of events or create particular effects (e.g., tension or suspense)
g. show internal mental processes to develop complex characters and convey their needs, motives, and emotional responses
h. provide an engaging conclusion, such as a surprise ending, a reflection, or a conclusion that returns to the beginning

Informative and Explanatory Texts

a. establish the topic in an introduction that provides a sense of what's to follow
b. develop the subject through relevant and specific facts, concrete details, quotations, or other information and examples
c. organize specific information under broader concepts or categories and provide headings, figures, tables, or diagrams when useful
d. use factual, precise language and maintain a formal, objective style when appropriate
e. use strategies appropriate to informational and explanatory texts such as defining, classifying, comparing/contrasting, and cause/effect
f. use appropriate links to join ideas and create cohesion
g. provide only accurate and relevant information
h. provide a conclusion that follows logically from the information or explanation presented

Arguments

a. introduce a claim about a topic or concept
b. support claims with logical reasons
c. support reasons with detailed and relevant evidence

[25] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

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d. signal the relationship between reasons, or between reasons and evidence, using words, phrases, and clauses (e.g., *another reason*, *such as*, *therefore*, *in addition*)
e. sustain an objective style and tone appropriate for making a case when appropriate to the discipline or context
f. include only relevant information and evidence in support of claims
g. provide a concluding statement or section that offers reflections, a restatement, or recommendations that follow from the argument

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively one on one, in small groups, and as a whole class, joining in discussions and remaining flexible and adaptable as participants.
2. Sustain concentration on information presented orally, visually, or multi-modally and confirm understanding by drawing well-supported inferences about the purpose and meaning of the information.

Standards — Students can and do (by key communication type):

Classroom discussions and collaboration

a. come to discussions having completed reading or other preparation in advance and draw on that material explicitly in discussions
b. determine a speaker's attitude or point of view toward a topic presented orally or through other media
c. ask questions to check understanding to clarify the main ideas and the supporting evidence of material presented orally or through other media
d. advance a discussion by answering questions precisely and sharing specific factual knowledge and observations supported by credible evidence
e. interpret information presented in visual and digital formats and explain how this data clarifies and contributes to a discussion or information presented orally
f. support productive teamwork by setting clear goals and deadlines, monitoring progress and participation of each team member, and taking different views into account and modifying own views when indicated in light of what others say

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Share experiences, opinions, and other information, gaining and maintaining the interest and response of listeners.
4. Use appropriate tone and phrasing for emphasis, demonstrating a growing command of formal English when indicated or appropriate (e.g., presenting ideas versus class discussion).

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. organize and present information about situations, topics, or texts that emphasize salient points and clarify and support claims and findings with pertinent and specific descriptions, facts, and examples in ways that are accessible and verifiable to listeners
b. use gesture, tone, phrasing, and pace for emphasis
c. incorporate visual displays and electronic media when helpful and in a manner that strengthens the presentation
d. perform dramatic readings of various prose and poetry speaking with clarity, fidelity, and responsiveness to the text, noting changes in the situation, mood, or tone of text

Language Development Standards

Conventions

In grades 6–8, students develop a firm command of sentence structure. They are able to form sentences of varying structures, place phrases and clauses properly within a sentence, and use a variety of coordinating and subordinating conjunctions to express relationships between sentence parts. Students have also mastered pronoun use, ensuring proper case, number, and person and avoiding vagueness. They understand and use verb voice and mood, and identify and correct inappropriate shifts in pronouns and verbs. Students set off nonrestrictive or parenthetical elements from the rest of the sentence with proper punctuation and use a comma before a coordinating conjunction in a compound sentence. They vary sentence patterns for effect and edit writing for redundancy and wordiness.

> Key Terms: conjunction; dash; nonrestrictive/parenthetical element; indicative, imperative, interrogative, conditional, and subjunctive mood; parentheses; phrase and clause; pronoun case, number, and person; simple, compound, complex, and compound-complex sentence; active and passive voice

Grammar and usage

Core Standards — Students can and do:

1. Form compound, complex, and compound-complex sentences.
2. Place phrases and clauses within a sentence, avoiding misplaced and dangling modifiers.*
3. Ensure that pronouns are in the proper case (subjective, objective, possessive).
4. Recognize and correct inappropriate shifts in pronoun number and person.*
5. Recognize and correct vague pronouns with unclear or ambiguous antecedents.*
6. Form and use verbs in the active and passive voice.
7. Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood.
8. Avoid inappropriate shifts in verb voice and mood.*

Mechanics

Core Standards — Students can and do:

9. Use punctuation to set off nonrestrictive/parenthetical elements with commas, parentheses, or dashes.*
10. Use a comma before a coordinating conjunction in a compound sentence.

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Word choice and style

Core Standards — Students can and do:

11. Use verbs in the active and passive voice and in the conditional and subjunctive moods to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).
12. Vary sentence patterns for meaning, reader/listener interest, and style.*
13. Choose words and phrases to express ideas precisely and concisely, avoiding wordiness and redundancy.*[26]

Grade-Level Focus
Focus by Grade-Level
Grade 6: Pronouns (Conventions Standards #s 3-5)
Grade 7: Sentence structure (Conventions Standards #1, #2, #12)
Grade 8: Verb voice and mood (Conventions Standards #s 6-8, #11)

Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In grades 6–8, students continue to make use of a range of strategies to determine and clarify the meaning of unknown and multiple-meaning words. This repertoire now includes considering the word's use in a broader context that includes the content of the paragraph in which the word appears and the overarching structure of the text. They habitually verify their inferences of word meanings. They interpret a variety of figurative language found in what they read. They make distinctions among words based on connotation. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly. This includes a continuing focus on "Tier 2" words and phrases (those that commonly appear in writing but not in spoken language), "Tier 3" words and phrases (those that are specific and important to particular disciplines).

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - using knowledge of roots, prefixes, and suffixes
   - using semantic clues, such as sentence and paragraph context as well as the organizational structure of the text (e.g., cause and effect, comparison and contrast)
   - using syntactic clues, such as using its position within the sentence as a guide to whether it is a subject, verb, or object
   - consulting reference materials, including glossaries, dictionaries, and thesauruses, both print and digital
2. Determine the relevant meaning of multiple-meaning words by using context.
3. Verify the preliminary determination of a word's meaning (e.g., by checking the inferred meaning in context or by looking up the word in a dictionary).
4. Interpret figurative language, including metaphors, similes, and idioms.

[26] Conventions standards noted with an asterisk (*) need to be revisited by students in subsequent grades. See Appendix A for a complete listing.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

5. Distinguish a word from other words with similar but not identical meanings (synonyms).

Acquiring vocabulary

Core Standards — Students can and do:

6. Acquire and use a grade-appropriate vocabulary of Tier 2 words taught directly and gained through reading.
7. Acquire and use a grade-appropriate vocabulary of Tier 3 words taught directly and gained through reading.

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Grade 6 English Language Arts: Focus for Instruction

Reading and Literature

In grade 6, students apply the reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read text at the 6-8 grade band level independently, with scaffolding likely required for texts at the high end of the range.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 100% text at the 6-8 grade band level

Writing and Research

In grade 6, students apply the writing standards to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter time frames, responding to specific sources by conveying the main ideas and key details of literary or informational sources at the 6-8 grade band level of complexity and content.

Speaking and Listening

In grade 6, students apply the core speaking and listening standards in different contexts.

- Speaking and listening Standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 6, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions

- Vocabulary standards applied to both reading, writing, speaking and listening
- **Grade-specific conventions focus:** Pronouns
  - Ensure that pronouns are in the proper case… (Conventions Standard #3)
  - Recognize and correct inappropriate shifts… (Conventions Standard #4)
  - Recognize and correct vague pronouns… (Conventions Standard #5)

Grade 7 English Language Arts: Focus for Instruction

Reading and Literature

In grade 7, students apply the reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read text at the 6-8 grade band level independently and are introduced to 9-10 grade band level "stretch" texts, which will likely require scaffolding.

- Reading Standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 90% at the 6-8 grade band level, 10% at the 9-10 grade band level

Writing and Research

In grade 7, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by analyzing the contents of literary or informational sources at the 6-8th grade band level of complexity and content.

Speaking and Listening

In grade 7, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 7, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Sentence structure
  - Form compound, complex… (Conventions Standard #1)
  - Place phrases and clauses… (Conventions Standard #2)
  - Vary sentence patterns… (Conventions Standard #12)

Grade 8 English Language Arts: Focus for Instruction

Reading and Literature

In grade 8, students apply the reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on reading text at the 6-8 grade band level independently as well as on sustained practice with 9-10 grade band level "stretch" texts, which may require scaffolding.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 70% at the 6-8 grade band level, 30% at the 9-10 grade band level

Writing and Research

In grade 8, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by analyzing the contents of literary or informational sources at the 6-8th grade band level of complexity and content.

Speaking and Listening

In grade 8, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 8, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Verb voice and mood
  - Form and use verbs in the active and passive voice (Conventions Standard #6)
  - Form and use verbs in the indicative… (Conventions Standard #7)
  - Avoid inappropriate shifts… (Conventions Standard #8)
  - Use verbs in the …voice …mood… (Conventions standard #11)

English Language Arts

Grades 9–10

Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grade

While advancing through grades 9–10, students must engage with texts of steadily increasing complexity.

- **In grade 9**, students focus on reading texts in the 9–10 grade band level with scaffolding likely required for texts at the high end of the range.
- **In grade 10**, students focus on reading texts in the 9–10 grade band level (70 percent) independently and are introduced to texts in the 11–CCR grade band level as "stretch" texts (30 percent), which will likely require scaffolding.

Determining Text Complexity for Grades 9–10[27]

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts	Quantitative Measures of Texts
• *Structure:* Implicit, subtle; graphic representations are essential to meaning; texts of increasing length • *Purpose:* Multiple; often implicit • *Style and Language:* Demanding; many literary devices; extensive use of Tier 2 and 3 words and figurative language • *Richness:* Several ideas/concepts; abstract • *Relationships:* Several connections; implicit • *Knowledge Demands:* Ability to handle challenging themes, consider multiple perspectives, and understand experiences distinctly different from one's own; cultural and historical knowledge useful for understanding characters, settings, and allusions; extensive discipline-specific content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences as well as their interests, motivations, and maturity level.	

[27] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for 9–10

Narratives	Drama	Poetry	Informational Text
At this level, includes the subgenres of adventure stories, biographies, memoirs, historical fiction, mysteries, science fiction, mysteries, myths, science fiction, realistic fiction, allegories, parodies, satire, and graphic novels.	*At this level, includes one-act and multi-act plays both in written form and on film.*	*At this level, includes the subgenres of narrative poems, lyrical poems, free verse, odes, ballads, and epics.*	*At this level, Includes such subgenres as exposition and argument in the form of essays, speeches, opinion pieces as well as other documents and digital media sources on a range of topics.*

Illustrative Texts for Narratives, Drama, and Poetry[28]

The Odyssey by Homer (8th century B.C.E.) translated by Robert Fagles

The Grapes of Wrath by John Steinbeck (1939)

The Killer Angels by Michael Shaara (1975)

In the Time of the Butterflies by Julia Alvarez (1994)

The Glass Menagerie by Tennessee Williams (1944)

"Song" by John Donne (1635)

"The Raven" by Edgar Allen Poe (1845)

"Loveliest of Trees" by A.E. Houseman (1896)

"I Am Offering This Poem to You" by Jimmy Santiago Baca (1977)

***Seminal historical texts that all students are expected to read**

Illustrative Informational Texts

"Second Inaugural Address" by Abraham Lincoln (1865)**

"State of the Union Address" by Franklin Delano Roosevelt (1941)

"Remarks to the Senate in Support of a Declaration of Conscience" by Margaret Chase Smith (1950)

"Address at the March on Washington" by Martin Luther King, Jr. (1963)**

"A Quilt of a Country" by Anna Quindlen (2001)

[28] See Appendix B for other texts illustrative of Grades 9–10 text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Read the text closely to determine what the text says explicitly and to make logical inferences from it; cite text evidence to support analyses in discussion and in writing.
2. Articulate the theses and themes and summarize how they develop over the course of the text and how they are expressed by the key details.
3. Analyze in detail how complex and multifaceted events, ideas, and characters unfold and interact over the course of the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. draw on specific details to describe how the events, characters, or setting develops over the course of the drama, narrative poem, or story
b. summarize the development of a theme and describe how that theme resonates throughout the text
c. weave together the details of texts to form a comprehensive understanding of its characters, including their overlapping or competing motivations
d. describe how the accumulation of specific phrases and images within poems contributes to a theme as a whole

Informational Text

a. demonstrate a command of the precise details of the exposition or argument, drawing on specific points to support an understanding of a part or the text as a whole
b. analyze the development of theses or explanations in texts and summarize succinctly the key relationships among ideas and supporting details

Observing craft and structure

Core Standards — Students can and do:

4. Interpret the meanings of words and phrases, including connotative and figurative meanings, and explain how specific word choices shape the meaning and tone of the text.
5. Analyze the structure of complex text and its parts, including how specific sentences, paragraphs, and larger portions build on each other and contribute to the whole of the text.
6. Compare and contrast the content and style of two or more texts written on similar topics or themes.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. analyze how the precise choice of words and phrases creates vivid images and sets the tone, mood, and theme of the text; compare the impact of words selected by the author to similar words with different connotations

b. explain how authors manipulate time (e.g., flashbacks, foreshadowing, pacing) to create suspense, mystery, or humor
c. evaluate how playwrights use soliloquies to portray the internal thinking and feeling of characters
d. compare and contrast similarities and differences in styles and forms of poems on a similar theme or topic

Informational Text

a. analyze how the author uses specific words and metaphors to establish tone or to make illuminating comparisons in an argument, explanation, or description
b. explain how the author structures information or an argument to emphasize key points and advance a point of view
c. analyze how different authors organize and categorize similar information and describe the impact of those different approaches

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Synthesize information presented graphically or visually in print, videos, or electronic texts with the information provided by the text.
8. Follow and evaluate the logic and reasoning of the text, including assessing whether the evidence provided is sufficient to support the claims.
9. Analyze the point of view or purpose represented in the text, assessing how it shapes the content, style, and tone.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. explain how a story unfolds when it is told by alternating or multiple narrators with different points of view
b. analyze literature in terms of its connection to related historical and cultural events and contexts

Informational Text

a. interpret complex, multifaceted, quantitative, or technical information presented in maps, charts, illustrations, graphs, and time lines
b. provide an account of an author's precise claims, including how specific assertions are defined and distinguished from opposing statements
c. analyze the explicit and implicit premises of an argument and determine if the conclusions reached are logically justified by the evidence presented in the text
d. compare how different authors construct and develop different points of views or perspectives on similar events or issues by assessing their assumptions, evidence, and reasoning

Developing habits for reading complex text

Core Standards — Students can and do:

10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding texts.

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Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write informative and explanatory texts and arguments that match purpose to task and address familiar as well as more distant, unknown and general audiences (e.g., peers, elected officials and policy makers, community members).

Conducting research

Core Standards — Students can and do:

2. Demonstrate proficiency at performing short, focused research projects as well as more sustained inquiries that demonstrate an increasing command of the subject under investigation.
3. Assemble evidence independently from authoritative and credible print and digital sources.
4. Assess the credibility, reliability, consistency, and accuracy of the information and sources gathered and determine the strengths and limitations of each source and avoiding over-reliance on any one source.
5. Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism.
6. Cite print or electronic sources correctly and document quotations, paraphrases, graphics, and other information using a standard format.

Revising writing

Core Standards — Students can and do:

7. Strengthen writing through revision, editing, or beginning again to ensure to ensure logical organization, precision of word choice, and coherence.

Using tools and technology

Core Standards — Students can and do:

8. Use technology and other tools to produce, revise, and distribute writing, as well as to interact online with others about writing, including responding to and providing feedback.

Developing proficiency in a range of writing

9. Create writing over extended timeframes (time for reflection and revision) and shorter timeframes (a single sitting or a day or two), responding to specific sources.

   Focus by grade level:

   Grade 9: Analyzing the content of literary or informational sources at the 9-10 grade band level of text complexity and content
   Grade 10: Comparing or evaluating the contents of literary or informational sources at the 9-10 grade band level of complexity and content

Standards — Students can and do (by key text type):[29]

Narratives

By high school, students are most often using narrative writing as a technique embedded within other genres. They use narrative writing to inform and persuade. They may, for example, provide a brief anecdote to support a point made in an argument or a scenario to illustrate an explanation. In such cases, narrative writing is a technique rather than a form in itself.

Informative and Explanatory Texts

a. provide a clear and coherent introduction that establishes the subject and conveys a knowledgeable stance
b. develop a complex subject through relevant and specific facts, concrete details, quotations, or other information and examples
c. organize complex information into categories that make clear distinctions and provide headings, figures, tables, and diagrams when useful
d. employ discipline-specific and technical vocabulary and maintain a formal, objective style
e. adapt strategies to present information and explanations (e.g., if/then, extended definitions, classification, comparison/contrast, and cause/effect) and employ them to manage the complexity of a topic
f. link ideas with transitions and by varying sentence structures to express relationships between ideas and create cohesion
g. emphasize the most significant information and confirm the accuracy of key points
h. provide a conclusion that articulates the implications and significance of the information or explanation

Arguments

a. establish a substantive claim and distinguish it from alternate or opposing claims
b. support claims with logical reasons
c. provide relevant and sufficient evidence from credible sources in support of the reasons
d. explain how the evidence links to the claim
e. develop the argument in part based on knowledge of the audience (e.g., building bridges by opening with areas of agreement)
f. convey relationships between reasons, as well as between reasons and evidence, and signal alternative claims using words, phrases, and clauses (e.g., *on the other hand*, *however*, *but*, *nevertheless*, *because, therefore, in addition*).
g. maintain a formal style when appropriate to the discipline or context
h. enhance the reliability of the argument by employing strategies such as paraphrasing or quoting explicitly from a credible, authoritative source
i. provide a concluding statement or section that enhances the argument, using strategies such as articulating the implications, summing up the key factors, or weighing the evidence to support the claim

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively in a range of structured interactions—both interpersonally and in groups—exchanging information constructively and with confidence.

[29] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

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2. Sustain concentration on complex information presented orally, visually, or multi-modally and confirm understanding by summarizing, analyzing, and elaborating on key ideas.

Standards — Students can and do (by key communication type):

Classroom discussions and collaboration

a. come to discussions having researched, studied, and taken notes on topics or issues under study and draw upon that preparation in discussions
b. determine the key ideas as well as the tone and mood of communications presented orally or through other media
c. ask questions to test the evidence that supports a speaker's claims and conclusions presented orally or through other media
d. build on essential information from others' input and respond constructively by making cogent and verifiable comments that aid in the furthering and deepening of discussions
e. integrate multiple streams of data presented through a variety of multi-modal media into a cohesive, meaningful understanding of the information
f. support productive teamwork by identifying the comments and claims made on all sides of an issue; evaluating the degree to which each claim is supported by evidence; sifting, summarizing, and putting to use the most important ideas developed by the group; and determining what additional information, research, and tasks are required in order to move the group towards its goals

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Present information and points of view, structuring and organizing comments to support their purposes and guide the listener.
4. Vary intonation and phrasing for emphasis and effect, demonstrating command of formal English when indicated or appropriate (e.g., presenting ideas versus class discussion).

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. organize and present complex information about situations, topics, or texts so that listeners can follow the line of thought by grouping related ideas, using transitional markers, and clarifying one's claims with evidence that is verifiable and accessible
b. align verbal (tone, phrasing, pacing) and nonverbal strategies (gestures and facial expressions) for emphasis and effect
c. make strategic use of multimedia elements and visual displays of data to enhance understanding
d. perform dramatic readings of various prose and poetry, speaking with clarity, fidelity, and responsiveness to the text, reflecting on syntax and diction for cues regarding emphasis and rhythm

Language Development Standards

Conventions

In high school, students gain a broad range of sophisticated language skills to enhance meaning, achieve stylistic effect, and create subtle links between and among ideas. They maintain parallel structure. They acquire a more conceptual understanding of usage and the limits of "rules." They use a full range of punctuation, including ellipses, semicolons,

colons, and hyphens, and have a fuller understanding of how to employ commas and dashes. They make use of a wide range of phrases and clauses for effect. They maintain a consistent style and tone, using a style manual appropriate to the discipline in which they are working to help conventionalize their writing.

Key Terms: colon, ellipses, hyphen, semicolon, parallel structure, verbal

Grammar and usage

Core Standards — Students can and do:

1. Use parallel structure in writing.
2. Consult references (e.g., *Merriam-Webster's Dictionary of English Usage*) as needed to resolve particular usage issues, particularly when the usage is contested.

Mechanics

Core Standards — Students can and do:

3. Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore a light*[,] *blue suit*).
4. Use a comma, ellipses, or dash to indicate a pause or break.
5. Use a semicolon (and perhaps a conjunctive adverb) to link two or more closely related independent clauses.
6. Use a colon to introduce a list or a quotation.
7. Observe the conventions concerning using hyphens to join words.

Word choice and style

Core Standards — Students can and do:

8. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to add variety and interest to writing.
9. Maintain consistency in style and tone.
10. Write and edit work so that it conforms to the guidelines in a style manual.

Focus by Grade-Level
Grade 9: Style (Conventions Standards #9, #10)
Grade 10: Advanced punctuation use (Conventions Standards #s 3-7)

Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In high school, students continue to make use of a range of strategies to determine and clarify the meaning of unknown and multiple-meaning words. This repertoire now includes considering multiple levels of context (sentence, paragraph, and text levels) and the word's history. They habitually verify their inferences of word meanings. They interpret a wide range of figurative language found in what they read and consider its contribution to the text. Possessing a highly developed sense of the shadings among words with similar denotations,

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they evaluate an author's or speaker's choice of words as well as alternatives to the words chosen. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly. This includes a continuing focus on "Tier 2" words and phrases (those that commonly appear in writing but not in spoken language), "Tier 3" words and phrases (those that are specific and important to particular disciplines).

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - using knowledge of roots, prefixes, and suffixes
   - using context, including syntactic and semantic clues, at the sentence, paragraph, and text levels
   - consulting reference materials, including general and specialized dictionaries and thesauruses, both print and digital
2. Determine the relevant meaning of multiple-meaning words by using context.
3. Verify the preliminary determination of a word's meaning (e.g., by checking the inferred meaning in context or by looking up the word in a dictionary).
4. Interpret figurative language and analyze its role within the text.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

5. Assess and explain the merits of the choice of one word over another in reading, writing, speaking, and listening.
6. Gain a clearer sense of a word's meaning and use by comparing it to other words with similar but not identical meanings (synonyms).

Acquiring vocabulary

Core Standards — Students can and do:

7. Acquire and use a grade-appropriate vocabulary of Tier 2 words taught directly and gained through reading.
8. Acquire and use a grade-appropriate vocabulary of Tier 3 words taught directly and gained through reading.

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Grade 9 English Language Arts: Focus for Instruction

Reading and Literature

In grade 9, students apply the core reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read 9-10 grade band text independently, with scaffolding likely required for texts at the high end of the range.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 100% 9-10 Band Text

100%

Writing and Research

In grade 9, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus: Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by analyzing the contents of literary or informational sources at the 9-10th grade band level of complexity and content.

Speaking and Listening

In grade 9, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 9, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Style
  - Maintain consistency … (Conventions Standard #9)
  - (Style manual)… (Conventions Standard #10)

Grade 10 English Language Arts: Focus for Instruction

Reading and Literature

In grade 10, students apply the core reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read 9-10 grade band text independently as well as on sustained practice with 11-CCR band "stretch" texts, which will likely require scaffolding.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 70% 9-10 Band Text; 30% 11-CCR text

70%	30%

Writing and Research

In grade 10, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by analyzing the contents of literary or informational sources at the 9-10th grade band level of complexity and content.

Speaking and Listening

In grade 10, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 10, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Advanced Punctuation Use
  - Coordinate adjectives … (Conventions Standard #3)
  - Comma/dash/ellipsis… (Conventions Standard #4)
  - Semicolon … (Conventions Standard #5)
  - Colon … (Conventions Standard #6)
  - Hyphen … (Conventions Standard #7)

English Language Arts

Grades 11–CCR

Required Text Complexity by Grade

Proportion of Texts Within and Above Grade Band to be Read in Each Grad

Grades	9–10 Level Text	11–CCR Level Text	Beyond CCR
9	100%		
10	70%	30%	
11		100%	
12		70%	30%

While advancing through grades 11-12, students must engage with texts of steadily increasing complexity.

- **In grade 11**, students focus on reading texts in the 11-CCR grade band level with scaffolding likely required for texts at the high end of the range.
- **In grade 12**, students focus on reading texts in the 11-CCR grade band level (70 percent) independently and are introduced to texts in the "Beyond CCR" grade band level as "stretch" texts (30 percent), which will likely require scaffolding.

Determining Text Complexity for Grades 11–CCR[30]

Text complexity is determined by a mix of qualitative and quantitative measures of the text itself refined by teachers' professional judgment about the match of particular texts to particular students. The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Qualitative Measures of Texts	Quantitative Measures of Texts
• *Structure:* Implicit, complex, unconventional; sophisticated graphic representations are essential to meaning; texts are sufficiently long to address complex subjects • *Purpose:* Multiple; often implicit and may be hidden or obscure • *Style and Language:* Unfamiliar, demanding, complex; many literary devices; extensive use of Tier 2 and 3 words and figurative language; language may be intentionally or unintentionally ambiguous • *Richness:* Many ideas/concepts; highly abstract; high information density • *Relationships:* Many implicit, complex, interwoven connections • *Knowledge Demands:* Ability to handle one or more complex themes, consider multiple and unusual perspectives, and understand experiences distinctly different from one's own; cultural and historical knowledge useful for understanding characters, settings, and allusions; extensive, perhaps specialized discipline-specific content knowledge	A study is underway with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity.
Professional Judgment that weighs students' prior knowledge and life experiences, students' interests, motivations, and maturity level.	

[30] Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

Mix of Key Text Types for 11–CCR

Narratives	Drama	Poetry	Informational Text
At this level, includes the subgenres of adventure stories, biographies, memoirs, historical fiction, mysteries, science fiction, mysteries, myths, science fiction, realistic fiction, allegories, parodies, satire, and graphic novels.	*At this level, includes one-act and multi-act plays both in written form and on film.*	*At this level, includes the subgenres of narrative poems, lyrical poems, free verse, odes, ballads, and epics.*	*At this level, Includes such subgenres as exposition and argument in the form of essays, speeches, opinion pieces as well as other documents and digital media sources on a range of topics.*

Illustrative Texts for Narratives, Drama, and Poetry[31]

Pride and Prejudice by Jane Austen (1813)

Black Boy by Richard Wright (1945)

Their Eyes Were Watching God by Zora Neale Hurston (1937)

The Bluest Eye by Toni Morrison (1970)

The Namesake by Jhumpa Lahiri (2003)

The Importance of Being Earnest by Oscar Wilde (1895)

Death of a Salesman by Arthur Miller (1949)

"Ode on a Grecian Urn" by John Keats (1820)

"Because I Could Not Stop for Death" by Emily Dickinson (1890)

Illustrative Informational Texts

The Declaration of Independence by Thomas Jefferson (1776)**

The Crisis by Thomas Paine (1776)

Walden by Henry David Thoreau (1854)

"Politics and the English Language" by George Orwell (1946)

"Letter from a Birmingham Jail" by Martin Luther King (1963)**

"Mother Tongue" by Amy Tan (1990)

Seminal historical texts that all students are expected to read

[31] See Appendix B for other texts illustrative of Grades 11–CCR text complexity.

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Reading and Literature Standards

Grasping specific details and key ideas

Core Standards — Students can and do:

1. Read the text closely to determine what the text says explicitly and to make logical inferences from it; cite text evidence to defend and challenge analyses in discussion and in writing.
2. Articulate the text's theses and themes and provide a summary that clarifies the relationships among ideas and the connections between key details.
3. Analyze in detail how complex and multifaceted events, ideas, and characters unfold and influence one another over the course of the text.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. analyze where the author chooses to focus and which details the author chooses to emphasize
b. analyze how multiple themes and ideas in the text interact and build on one another
c. evaluate the extent to which setting shapes the course of events and sets the mood
d. trace the origins and evolution of the traits, motivations, and relationships among characters and how they interact to influence the plot and its resolution
e. describe how the poet develops a central image, preoccupation, or idea through the accumulation of specific phrases and images

Informational Text

a. demonstrate an understanding of the precise elements of an author's explanation or argument, including the distinctions the author makes between different ideas or information
b. scrutinize the details within specific portions of texts and connect the insights gained to develop an understanding of the text as a whole
c. analyze how the text captures the interaction between complex ideas or multifaceted events

Observing craft and structure

Core Standards — Students can and do:

4. Interpret the meanings of words and phrases, including connotative and figurative meanings, and analyze how word choices have a significant effect on the meaning and tone of the text.
5. Analyze the ways the author chooses to structure the text, including how to present complex ideas and events and where to begin and end.
6. Compare and contrast the choices different authors make in treating similar topics or themes, including content, style, and tone.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. analyze how the author's use of language impacts the text, including the degree of formality of the diction and how it is evocative of a particular setting (e.g., a courtroom, a rural town)
b. evaluate how authors create meaningful ambiguity and multiple layers of meaning in poetry, drama, and other narratives
c. analyze how an author choice of where to begin a story, poem, or drama impacts the overall plot structure
d. contrast alternative treatments of the same dramatic work in different stage productions and evaluate how the directors' different interpretations relate to evidence within the script
e. analyze how the author draws upon and transforms fictional or historical source material (e.g., how Shakespeare draws on Plutarch or a story in Ovid)

Informational Text

a. describe how the choice of a particular word, phrase, or series of words can impact significantly the meaning of a document (e.g., contract, court opinion, essay)
b. evaluate how the author's choice of structure contributes to the effectiveness of the exposition or argument
c. compare and contrast presentations of the same topic in different media and describe the differences in focus, organization, and links to other sources

Integrating information and evaluating evidence

Core Standards — Students can and do:

7. Synthesize information presented graphically or visually in print, videos, or electronic texts and, when appropriate, note discrepancies of fact or interpretation (e.g., data in a table inconsistent with the author's analysis).
8. Rigorously evaluate the logic and reasoning of the text, including assessing whether the evidence provided is relevant and sufficient.
9. Analyze how the point of view or purpose develops in the text and explain how it is revealed in the key details.

Standards — Students can and do (by key text type):

Narratives, Drama, and Poetry

a. compare points of view from which different stories are told and trace how they shift within a story and influence characterization and plot
b. explain how dramatic irony created by the differences between what the audience or reader knows and what the characters know in a drama or narrative fiction creates suspense, anxiety, or humor

Informational Text

a. synthesize ideas and data presented graphically and determine their purpose and relationship to the rest of the text (print or digital), noting any inconsistencies or discrepancies between the two
b. evaluate the reasoning and rhetoric that support an argument or explanation, including assessing the sufficiency and relevance of the evidence as well as identifying any unsubstantiated statements or fallacious reasoning
c. analyze documents of historical and literary significance for their premises, perspectives, and logical structure

Developing habits for reading complex text

Core Standards — Students can and do:

10. Develop the habit of reading independently and productively, sustaining concentration and stamina to read increasingly demanding texts.

Writing and Research Standards

Writing to reflect audience, purpose, and task

Core Standards — Students can and do:

1. Write informative and explanatory texts and arguments that match purpose to task and are tailored to audiences with specific requirements (e.g., admissions officer, human resources officer, skeptical audience).

Conducting research

Core Standards — Students can and do:

2. Demonstrate proficiency at performing short, focused research projects as well as more sustained inquiries that synthesize multiple authoritative sources on a subject.
3. Analyze evidence independently gathered from multiple authoritative and credible print and digital sources.
4. Assess the credibility, reliability, consistency, and accuracy of the information and sources gathered and determine their usefulness and relevance for the specific audience, purpose, and task.
5. Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism.
6. Cite print or electronic sources correctly and document quotations, paraphrases, graphics, and other information using a standard format.

Revising writing

Core Standards — Students can and do:

7. Strengthen writing through revision, editing, or beginning again to ensure to ensure logical organization, precision of word choice, and coherence.

Using tools and technology

Core Standards — Students can and do:

8. Demonstrate command of technology and other tools to produce, revise, and distribute writing, as well as to interact online with others about writing, including responding to and providing feedback.

Developing proficiency in a range of writing

9. Create writing over extended timeframes (time for reflection and revision) and shorter timeframes (a single sitting or a day or two), responding to specific sources.

   Focus by grade level:

   Grade 11: Analyzing the content of literary or informational sources at the 11-CCR grade band level of text complexity and content

   Grade 12: Synthesizing or evaluating the contents of literary or informational sources at the 11-CCR grade band level of complexity and content

Standards — Students can and do (by key text type):[32]

Narratives

By high school, students are most often using narrative writing as a technique embedded within other genres. They use narrative writing to inform and persuade. They may, for example, provide a brief anecdote to support a point made in an argument or a scenario to illustrate an explanation. In such cases, narrative writing is a technique rather than a form in itself.

Informative and Explanatory Texts

a. provide a clear and coherent introduction that establishes the subject and conveys a knowledgeable stance
b. develop complex subjects through judicious use of relevant and specific facts, details, quotations, examples, or other information
c. organize and present information so that each new piece of information builds upon what precedes it to create a unified whole
d. demonstrate command of discipline-specific and technical vocabulary when appropriate and adjust style as appropriate to the situation
e. demonstrate control of a range of strategies to present complex information or explanations and employ them effectively to manage the complexity of the topic and accomplish the writer's purpose
f. link ideas with transitions and by varying sentence structures to express the precise relationships among ideas and create cohesion
g. provide a conclusion that articulates the implications and significance of the information or explanation

Arguments

a. establish the importance of the issue, make a substantive claim, and distinguish it from alternate or opposing claims
b. support claims with logical reasons
c. provide relevant, sufficient, and convincing evidence from credible sources in support of the reasons
d. make logical connections between the evidence and the claim
e. develop the argument in part based on an awareness of the audience's values, knowledge of the issue, and possible biases
f. convey relationships between reasons, as well as between reasons and evidence, and signal alternative claims using words, phrases and clauses (e.g., *on the other hand*, *however*, *but*, *nevertheless*, *because*, *therefore*, *in addition*)
g. maintain a formal style when appropriate to the discipline or context
h. enhance the credibility of the argument by demonstrating control of strategies, including paraphrasing or quoting from authoritative sources and citing logical consequences
i. provide a concluding statement or section that enhances the argument, using strategies such as articulating the implications, summing up the key factors, or weighing the evidence to support the claim

[32] See Appendix C for samples of student writing that illustrate through annotations the level of quality required to meet the writing standards.

Speaking and Listening Standards

Listening closely and participating productively

Core Standards — Students can and do:

1. Participate productively in a range of structured interactions—both interpersonally and in groups—exchanging information constructively and with confidence, adapting to different levels of formality.
2. Sustain concentration on complex information presented orally, visually, or multi-modally and confirm understanding by challenging or defending key ideas and supporting evidence.

Standards — Students can and do (by key communication type):

Classroom discussions and collaboration

a. come to discussions having formulated considered judgments on the topics or issues under study and draw upon that preparation in discussions
b. evaluate the content and rhetoric of a speaker, noting when evidence is exaggerated or distorted
c. ask questions that probe the reasoning and evidence that support the claims and conclusions made orally or through other media, including offering counter examples or other points of view
d. propel conversations forward by providing essential information and sharing findings that clarify, accommodate, or challenge ideas
e. synthesize information presented visually or digitally with other information presented orally, noting the effect on meaning of any discrepancies between the two presentations
f. assist in the formulation and productive functioning of both formal and informal self-directed work groups by identifying and assigning tasks and maintaining conversational norms as well as evaluating the progress of the team towards its goals

Exchanging information and speaking effectively

Core Standards — Students can and do:

3. Present information clearly and persuasively to others, selecting the most appropriate way to structure comments for clarity and effect.
4. Adapt delivery, tone, and mood for emphasis and effect, demonstrating command of formal English when indicated or appropriate (e.g., presenting ideas versus class discussion).

Standards — Students can and do (by key communication type):

Presentation of ideas and information

a. organize and present complex information about topics, situations, or texts, providing reliable and credible evidence from authoritative sources in support of findings and claims such that the line of reasoning is clear and alternative perspectives are addressed
b. shape delivery and message to the occasion and the audience's values, knowledge of the issue, and possible biases
c. engage an audience and improve comprehension through visual aids in presentations, including multimedia platforms
d. portray and explain various ways to perform dramatic readings of various prose and poetry, citing text evidence for the alternative readings

Language Development Standards

Conventions

In high school, students gain a broad range of sophisticated language skills to enhance meaning, achieve stylistic effect, and create subtle links between and among ideas. They maintain parallel structure. They acquire a more conceptual understanding of usage and the limits of "rules." They use a full range of punctuation, including ellipses, semicolons, colons, and hyphens, and have a fuller understanding of how to employ commas and dashes. They make use of a wide range of phrases and clauses for effect. They maintain a consistent style and tone, using a style manual appropriate to the discipline in which they are working to help conventionalize their writing.

Key Terms: colon, ellipses, hyphen, semicolon, parallel structure, verbal

Grammar and usage

Core Standards — Students can and do:

11. Use parallel structure in writing.
12. Consult references (e.g., *Merriam-Webster's Dictionary of English Usage*) as needed to resolve particular usage issues, particularly when the usage is contested.

Mechanics

Core Standards — Students can and do:

13. Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore a light*[,] *blue suit*).
14. Use a comma, ellipses, or dash to indicate a pause or break.
15. Use a semicolon (and perhaps a conjunctive adverb) to link two or more closely related independent clauses.
16. Use a colon to introduce a list or a quotation.
17. Observe the conventions concerning using hyphens to join words.

Word choice and style

Core Standards — Students can and do:

18. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to add variety and interest to writing.
19. Maintain consistency in style and tone.
20. Write and edit work so that it conforms to the guidelines in a style manual.

Focus by Grade-Level
Grade 11: Parallel structure and phrasing (Conventions Standards #1, #8)
Grade 12: Usage (Conventions Standard #2)

Vocabulary

Key to students' vocabulary development is building rich and flexible word knowledge marked by multiple connections that link a word to similar words and to contexts and experiences that are related to that word—as compared to simply a definition. In high school, students continue to make use of a range of strategies to determine and clarify the meaning of unknown and multiple-meaning words. This repertoire now includes considering multiple levels of context (sentence, paragraph, and text levels) and the word's history. They habitually verify their inferences of word meanings. They interpret a wide range of figurative language found in what they read and consider its contribution to the text. Possessing a highly developed sense of the shadings among words with similar denotations, they evaluate an author's or speaker's choice of words as well as alternatives to the words chosen. They acquire new words through interactive language use, including informal talk, discussion, reading and responding to text as well as by being taught the words directly. This includes a continuing focus on "Tier 2" words and phrases (those that commonly appear in writing but not in spoken language), "Tier 3" words and phrases (those that are specific and important to particular disciplines).

Determining the meaning of words

Core Standards — Students can and do:

1. Determine or clarify the meaning of an unknown word by using one or more of the following strategies:
   - using knowledge of roots, prefixes, and suffixes
   - using context, including syntactic and semantic clues, at the sentence, paragraph, and text levels
   - consulting reference materials, including general and specialized dictionaries and thesauruses, both print and digital
   - using the word's history (etymology)
2. Determine the relevant meaning of multiple-meaning words by using context.
3. Verify the preliminary determination of a word's meaning (e.g., by checking the inferred meaning in context or by looking up the word in a dictionary).
4. Interpret figurative language and analyze its role within the text.

Understanding the nuances of words (denotations and connotations)

Core Standards — Students can and do:

7. Assess and explain the merits of the choice of one word over another in reading, writing, speaking, and listening.
8. Gain a clearer sense of a word's meaning and use by comparing it to other words with similar but not identical meanings (synonyms).

Acquiring vocabulary

Core Standards — Students can and do:

7. Acquire and use an extensive vocabulary of Tier 2 words taught directly and gained through reading.
8. Acquire and use a grade-appropriate vocabulary of Tier 3 words taught directly and gained through reading.

Grade 11 English Language Arts: Focus for Instruction

Reading and Literature

In grade 11, students apply the core reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read 11-CCR grade band text independently, with scaffolding likely required for texts at the high end of the range.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 100% 11-CCR Band Text

100%

Writing and Research

In grade 11, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter timeframes, responding to specific sources by analyzing the contents of literary or informational sources at the 11-CCR grade band level of complexity and content.

Speaking and Listening

In grade 11, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 11, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Parallel Structure and Phrasing
  - Use parallel structure in writing … (Conventions Standard #1)
  - Use various types of phrases… (Conventions Standard #8)

Grade 12 English Language Arts: Focus for Instruction

Reading and Literature

In grade 12, students apply the core reading standards to the following types of text: narratives, drama, poetry, and informational text. Students focus on learning to read 11-CCR grade band text independently as well as on sustained practice with Beyond CCR band "stretch" texts, which will likely require scaffolding.

- Reading standards applied to different text types
- Mix of text types: Narratives, Drama, Poetry, Informational Text
- **Text Complexity focus:** 70% 11-CCR Band Text; 30% Beyond CCR text

Writing and Research

In grade 12, students apply the standards in writing to the following types of text: Narrative, Informative/Explanatory, and Argument. Students perform research, including short focused research tasks. They also write over various time frames in response to specific sources.

- Writing standards applied to different text types: Narrative, Informative/Explanatory, Argument
- Research, including short focused research tasks
- **Grade-specific focus:** Students create writing over extended and shorter time frames, responding to specific sources by synthesizing or evaluating the contents of literary of informational sources of 11-CCR grade band level complexity and content.

Speaking and Listening

In grade 12, students apply the core speaking and listening standards in different contexts.

- Speaking and listening standards applied in different contexts: classroom discussion and collaboration as well as in presentations of ideas and information.

Language Development

In grade 12, students apply the language development standards by applying the core vocabulary standards to determine word meaning, understand word nuances, and acquire vocabulary and to produce writing and speaking that observes appropriate conventions.

- Vocabulary standards applied to reading, writing, speaking and listening
- **Grade-specific conventions focus:** Usage
  - Consult references … (Conventions Standard #2)

APPENDIX A

Text Complexity Next Steps

A key requirement of the Core Standards in Reading is that all students engage with texts of steadily increasing complexity as they advance through school. The Core Standards' model of *text complexity*—in the simplest terms, how easy or difficult a text is to read—blends qualitative and quantitative measures of inherent text difficulty with educators' knowledge of their students. All three elements should be considered together when evaluating a text's appropriateness for particular students.

Qualitative dimensions are aspects of text best measured by readers applying trained judgment to the evaluation task. These dimensions include the text's structure, format, and length; its purpose; its style and language; the quality, nature, and density of its ideas, concepts, and information; relationships among ideas, information, and characters in it; and the knowledge and experience demands it places upon readers.

Quantitative dimensions include not only those aspects of text traditionally measured by readability formulas—word length and sentence length—but also computer-assessable aspects of text cohesion. These include referential cohesion (the degree to which a text refers back to previous points) and word frequency.

The qualitative and quantitative measures of a text are balanced in the model by educators' *professional judgment* of the appropriateness of the text for particular students given their background knowledge, interests, and motivation. Harder texts may be appropriate for highly knowledgeable or motivated students, and easier texts may be suitable as a means for building struggling readers' skills up to required levels.

While the tools included in this draft and the forthcoming ones described below represent an important advance over those previously available, no measure or set of measures is perfectly accurate. The mandate is that the body of works that students study in a given year represent an appropriate level of complexity as defined by these standards.

Current and next steps

A qualitative rubric, derived from prior studies and refined through feedback from trained teacher-raters, is included in this draft to define some ways in which text complexity should increase as students move through the grades. The rubric can be used (in conjunction with forthcoming quantitative measures) to place individual texts into grade bands by complexity. The qualitative dimensions are best understood, however, as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

The Core Standards work team is presently conducting a study with Coh-Metrix, a nonprofit research organization, to identify roughly five to seven computer-measurable dimensions of text cohesion. These dimensions, paired with a Lexile score, will yield a robust quantitative assessment of text complexity that, along with both the qualitative dimensions and professional judgment, will round out the Core Standards model of complexity. Graphically, these three elements will appear together in a "label" defining complexity for a given text.

Following the completion of that study in early 2010, the work team will oversee the development of a Web site designed to make the text complexity tools more user-friendly and broadly available. The site will contain a database of complexity information for a range of widely used texts, including links to texts and test passages of similar complexity. Educators will be able to input additional texts for evaluation and comment on the suitability of particular texts for particular groups of students. The overarching goal is to make text complexity a vital and easy-to-incorporate element of reading instruction.

Text Complexity Qualitative Scheme

Dimension of Text		Grade Span 2–3	Grade Span 4–5	Grade Span 6–8	Grade Span 9–10	Grade Span 11–12
Structure		Explicit, simple, conventional; simple graphic representations are supplementary to meaning; texts are relatively short	Largely explicit and direct; graphic representations are supplementary to meaning; texts are of increasing length	Largely implicit and subtle; graphic representations are essential to meaning; texts are of increasing length	Implicit, subtle; graphic representations are essential to meaning; texts are of increasing length	Implicit, complex, unconventional; sophisticated graphic representations are essential to meaning; texts are sufficiently long to address complex subjects
Purpose		Single; explicitly stated	Single or twofold; clearly indicated	Single or multiple; subtly stated	Multiple; often implicit	Multiple; often implicit and may be hidden or obscure
Style and Language	Style	Familiar, accessible, plain; few literary devices	Moderately accessible; some literary devices	Moderately demanding; several literary devices	Demanding; many literary devices	Unfamiliar, demanding, complex; many literary devices
	Language	Mostly clear, everyday language; limited use of Tier 2 and 3 words and figurative language	Some everyday language; some use of Tier 2 and 3 words and figurative language	Consistent use of Tier 2 and 3 words and figurative language	Extensive use of Tier 2 and 3 words and figurative language	Extensive use of Tier 2 and 3 words and figurative language; language may be intentionally or unintentionally ambiguous
Richness		A few ideas/concepts; concrete; low information density	Some ideas/concepts; mostly concrete; moderate information density	Several ideas/concepts; mostly abstract; moderate information density	Several ideas/concepts; abstract; high information density	Many ideas/concepts; highly abstract; high information density
Relationships		A few connections; explicit; simple	Some connections; largely explicit	Several connections; largely implicit	Several connections; implicit	Many connections; implicit; complex and interwoven
Knowledge Demands	Life experiences (literary texts)	Ability to handle simple themes and fantastical elements as well as draw upon common, everyday experiences	Ability to handle fairly simple themes, consider a perspective somewhat different from one's own, and understand unfamiliar experiences	Ability to handle fairly challenging themes, consider multiple perspectives, and understand unfamiliar experiences	Ability to handle challenging themes, consider multiple perspectives, and understand experiences distinctly different from one's own	Ability to handle one or more complex themes, consider multiple and unusual perspectives, and understand experiences distinctly different from one's own
	Cultural/literary knowledge (chiefly literary texts)	General background knowledge and familiarity with genre conventions required		Cultural and historical knowledge useful for understanding characters, settings, and allusions		
	Content/discipline knowledge (chiefly informational texts)	Some everyday and general content knowledge	Some general and discipline-specific content knowledge	Some discipline-specific content knowledge	Extensive discipline-specific content knowledge	Extensive, perhaps specialized discipline-specific content knowledge

Adapted from ACT, Inc., (2005); Carnegie Council on Advancing Adolescent Literacy (2010); Chall, Bissex, Conrad, & Harris-Sharples (1996); and Hess and Biggam (2004)

The qualitative dimensions of text complexity are best understood as continua of increasing complexity rather than as representing discrete and easily defined stages. Most authentic texts will exhibit some but not all of the traits linked to a particular grade band; qualitatively assigning a text to a grade band is therefore a matter of "best fit," or determining which grade band's set of descriptors most accurately describes the text.

Structure

- Explicit, simple, conventional → Implicit, complex, unconventional
- Simple graphic representations → Sophisticated graphic representations
- Graphic representations supplementary to meaning → Graphic representations essential to meaning
- Relatively short texts → Texts sufficiently long to address complex subjects

Purpose

- Single purpose → Multiple purposes
- Explicitly stated → Often implicit and may be hidden or obscure

Style

- Familiar, accessible, plain → Unfamiliar, demanding, complex
- Few literary devices → Many literary devices

Language

- Mostly everyday language → Extensive use of Tier 2 and 3 words
- Limited use of figurative language → Extensive use of figurative language
- Clear language → Potentially ambiguous language

Richness

- A few ideas/concepts → Many ideas/concepts
- Concrete ideas/concepts → Abstract ideas/concepts
- Low information density → High information density

Relationships

- A few connections → Many connections
- Explicit connections → Implicit connections
- Simple connections → Complex, interwoven connections

Knowledge Demands: Life Experiences

- Simple themes → Complex themes
- Single theme → Multiple themes
- Common, everyday experiences and fantastical elements → Experiences distinctly different from one's own
- Single perspective like one's own → Multiple and unusual perspectives

Knowledge Demands: Cultural Knowledge

- General background knowledge and familiarity with genre conventions required → Cultural and historical knowledge useful

Knowledge Demands: Content/Discipline Knowledge

- Some everyday and general content knowledge → Extensive, perhaps specialized discipline-specific content knowledge

Definitions of Key Writing Types

Narrative

Narrative writing is organized by time. Time is central because narrative writing depicts events, whether real or imagined. Narrative writing is fundamental to novels, short stories, biographies, autobiographies, historical accounts, and plays. With practice, students' repertoire of narrative strategies expands and their control of them increases. Students learn to provide visual details of scenes, objects, or people; to depict specific actions (movements, gestures, postures, and expressions); to use dialogue and interior monologue in order to provide insight into the narrator's and characters' personalities and motives; and to manipulate pace in order to highlight the significance of certain events and create tension and suspense. Narrative writing serves a variety of purposes; frequently it is embedded in other kinds of writing, such as writing intended to inform, instruct, or persuade.

Informative/Explanatory Text

Informative/explanatory writing conveys information accurately. This kind of writing can serve one or more of several closely related purposes: to increase readers' knowledge of a subject, to help readers better understand a procedure or process, or to enhance readers' comprehension of a concept. Informative/explanatory writing addresses questions, such as questions about types (What are the different types of whales?), about components (What are the parts of a motor?), about aspects of a subject such as its size, function, or behavior (How big is the United States? What is an x-ray used for? How do penguins find food?), about how things work (How does a camera work?), and about why things happen (Why is Earth warming?). To produce this kind of writing, students draw on what they already know and on primary and secondary sources. With practice, students become better able to develop a controlling idea that supports coherence and focus, and they can select examples, facts, and details that are relevant. They are also able to employ a variety of techniques that writers use to convey information, such as naming, describing, or differentiating different types or parts; comparing or contrasting one subject with another; and relating an anecdote or scenario to illustrate a point.

Argument

The purpose of argument is to persuade in order to change the reader's point of view or to bring about some action on the reader's part. There are many techniques employed by writers to persuade readers—for example, appeals to emotions, appeals to common beliefs, and the creation of a believable authorial voice. However, the core of argument is logic and evidence. A logical argument convinces its audience of the merit and reasonableness of the claims and the proof offered in support of the claims. Writers of logical arguments provide credible evidence (facts and details) to support their assertions. Although young children are not able to produce fully developed logical arguments, they are developing a variety of ways to extend and elaborate their work around opinions or judgments. They provide examples, they offer reasons for their assertions, and they explain cause and effect. These kinds of expository structures are steps on the road to argument.

Conventions

Three goals undergird the Conventions standards:

(1) Students should have a carefully specified range of broadly useful terms in order to be precise in their discussions about language. Such key terms (noted below) should be defined in grade-appropriate ways for younger students and fleshed out more fully in later grades. (For guidance on this matter, see, for example, Brock Haussamen with Amy Benjamin, Martha Kolln, and Rebecca S. Wheeler, *Grammar Alive!: A Guide for Teachers* [Urbana, IL: NCTE, 2003] and Amy Benjamin with Tom Oliva, *Engaging Grammar: Practical Advice for Real Classrooms* [Urbana, IL: NCTE, 2007].) Additional terminology may be helpful in particular instructional situations; avoiding terminology altogether may be appropriate in others.

(2) Students must be able to observe the conventions of standard English in their formal writing and speaking for the sake of having their efforts widely understood and taken seriously.

(3) Students need to understand that effective language use is more than simply observing a series of rules but also about making careful choices among alternatives, considering those choices in relation to task, purpose, audience, occasion, and discipline.

Many conventions-related issues are likely to arise in students' writing and speaking prior to their formal appearance in the sequence below. For example, students in kindergarten are expected to know what a complete sentence is even though the concept of a fragment is not mentioned specifically in the standards until grade 3.

Conversely, many skills and understandings introduced at lower grades will require continued attention as students advance in the grades. Students in grade 3, for instance, can ensure subject-verb agreement in simple situations, such as when the subject and verb appear next to each other in a sentence. As students' writing and speaking become more complex, however, new agreement challenges arise, such as intervening phrases suggesting a different number for the verb than the subject calls for. "Errors" with applying previously mastered skills and understandings are thus often a sign of progress in that students are stretching their ability to communicate. "Relearning" is then a matter of students becoming able to apply old skills and understandings in new, more sophisticated ways.

While all the Conventions standards should be considered cumulative, certain ones, noted with an asterisk (*), are particularly likely to need to be revisited by older students as they convey ever more elaborate ideas in writing and speech.

ELA Conventions Progressive Skills: By Standard

The following standards, marked with an asterisk (*) in the standards document, are skills and understandings that require continued attention in higher grades (after their introduction in lower grades) as they are applied to increasingly sophisticated writing and speaking.

Grade 3	Grades 4–5	Grades 6–8	(Grade/band in which the standard is introduced)
3.1 Generate complete sentences, avoiding sentence fragments, comma splices, and run-ons.			
3.2 Ensure subject-verb and pronoun-antecedent agreement.			
3.7 Choose words for effect.			
	4–5.2 Recognize and correct inappropriate shifts in verb tense.		
	4–5.3 Form and choose between adjectives and adverbs (including comparative and superlative forms), placing them appropriately within the sentence.		
	4–5.4 Correctly use frequently confused words.		
	4–5.5 Use idiomatic language.		
	4–5.7 Use punctuation to separate items in a series.		
	4–5.11 Spell grade-appropriate words correctly, consulting references as needed.		
	4–5.12 Use specialized, topic-specific language to convey ideas precisely.		
	4–5.13 Use figurative language to create images or make comparisons and connections between people, objects, or ideas.		
	4–5.14 Use punctuation for effect.		
	4–5.15 Expand, combine, and reduce sentences for meaning, reader/listener interest, and style.		
		6–8.2 Place phrases and clauses within a sentence, avoiding misplaced and dangling modifiers.	
		6–8.4 Recognize and correct inappropriate shifts in pronoun number and person.	
		6.8.5 Recognize and correct vague pronouns with unclear or ambiguous antecedents.	
		6–8.8 Recognize and correct inappropriate shifts in verb voice and mood.	
		6–8.9 Set off nonrestrictive/parenthetical elements with commas, parentheses, or dashes.	
		6–8.12 Vary sentence patterns for meaning, reader/listener interest, and style.	
		6–8.13 Choose words and phrases to express ideas precisely and concisely, avoiding redundancy and wordiness.	

Vocabulary Instruction

> Words are not just words. They are the nexus — the interface — between communication and thought. When we read, it is through words that we build, refine, and modify our knowledge. What makes vocabulary valuable and important is not the words themselves so much as the understandings they afford.
>
> Marilyn Adams[33]

The importance of students acquiring a rich and varied vocabulary cannot be overstated. Research suggests that if students are going to grasp and retain words and comprehend text, they need incremental, repeated exposure to words they are trying to learn in a variety of contexts. When students make multiple connections between new words and their own experiences they develop a nuanced and flexible understanding of the word. In this way, students learn not only what a word means, but how to use that word in a variety of contexts and apply appropriate senses of the word's meaning in order to understand different contexts.[34]

Initially children readily learn words from oral conversation because oral conversations are context rich in ways that aid in vocabulary acquisition: in discussion a small set of words (accompanied by gesture and intonation) is used with great frequency to talk about a narrow range of situations children are exposed to on a day to day basis. Yet as children reach school age, new words are less frequently introduced in conversation, and consequently vocabulary acquisition eventually stagnates by fourth or fifth grade unless students acquire additional words from written context.[35]

Written language, by contrast, contains hundreds of times as many different words as are typically used in conversational language. Yet writing lacks the interactive opportunities and nonverbal context provided by oral conversation so it presents a special challenge towards successful vocabulary acquisition without purposeful and ongoing concentration on vocabulary.[36] In fact, at most, between five and fifteen percent of new words encountered when reading are retained.[37] The weaker a student's vocabulary is, the slighter the gain.[38] Yet research shows that if students are going to understand what they read, they must understand upward of 95 percent of the words.[39]

As this "tipping point" for lexical dexterity is quite challenging for students to reach, every classroom needs to focus on providing students with high quality contextual encounters with vocabulary words that epitomize what they encounter in written texts. The aim should be to expose students to words that have the widest application—concepts that students are likely to meet again and again not just in classroom settings but outside the school walls as well. Some of these highly transferable academic words, often referred to as Tier 2 words, such as qualifying adjectives and adverbs (e.g., important, typically) are used broadly across domains and indeed in contexts that transcend the classroom.[40] However, the meanings of most words are specific to their domains—often referred to as Tier 3 words—including those that arise in multiple domains (e.g., chemical constituents, constituent voting patterns). To learn words, students have to read multiple selections from multiple authors within key domains of learning.

The problem is that, in any given instance, it is not the entire spectrum of a word's history, meanings, usages, and features that matters, but only those aspects that are relevant to the surrounding context. That means, first, that the reader's internal representation of the word must be sufficiently complete and well-articulated so that the intended meaning is available and, second, that the reader must understand the context well enough to select the intended meaning – which, in turn, depends on good understanding of the surrounding words of the passage.

Key to students' vocabulary development is building rich and flexible word knowledge. Students need plentiful opportunities to use and respond to the words they learn, through playful informal talk, discussion, and reading or being read to and responding to what is read. Along with attention to academic (Tier 2 words) and content-specific words (Tier 3 words), students benefit from instruction about the connections and patterns in language. Developing in students an analytical attitude toward the logic and sentence structure of their texts alongside an awareness of word parts, word origins, and word relationships provides students with a sense of how language works so that syntax, morphology and etymology can become useful cues to word in building meaning as students encounter new words and concepts in their reading.[41] As students are exposed to and interact with language throughout their school careers, they are able to acquire understandings of word meanings, build awareness of the workings of language, and apply word meanings to comprehend and produce language.

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[33] Adams, M. (2009). "The Challenge of Advanced Texts: The Interdependence of Reading and Learning," in Hiebert (Ed.), Reading more, reading better: Are American students reading enough of the right stuff?, New York: Guilford Publications.

[34] Landauer, TK, McNamara, DS, Dennis, S and Kintsch, W (2007) Handbook of Latent Semantic Analysis; Landauer, T. K., & Dumais, S. T. (1997). A solution to Plato's problem: The latent semantic analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review, 104(2)*, 211-240; Nagy, W. E., Herman, P., & Anderson, R. C. (1985). Learning words from context. *Reading* Research Quarterly, 20, 233-253.

[35] Hayes, D and Ahrens, M: "Vocabulary simplification for children: A special case of "motherese?" Journal of Child Language. Vol 15(2), Jun 1988, 395-410

[36] Ibid.

[37] Ibid.

[38] Daneman & Green, 1986; Herman, Anderson, Person &Nagy, 1987; Sternberg & Powell

[39] Betts, E. A. (1946). *Foundations of reading instruction.* New York, NY: American Book Company; Carver, R. P. (1994). Percentage of unknown vocabulary words in text as a function of the relative difficulty of the text: Implications for instruction. *Journal of Reading Behavior, 26,* 413-437; Hu, M., & Nation, P. (2000). Unknown vocabulary density and reading comprehension. *Reading in a Foreign Language, 13(1)*, 403-430; Laufer, B. (1988). What percentage of text-lexis is essential for comprehension. In C. Lauren & M. Nordmann (Eds.), *Special language: From humans to thinking machines,* pp. 316-323. Clevedon, UK: Multilingual Matters.

[40] Indeed, the fact that these words transcend specific disciplines argues for them being taught and used across the curriculum by all teachers.

[41] Beck, I. L., McKeown, M. & Kucan, L. (2008). *Creating robust vocabulary: Frequently asked questions and extended examples.*

APPENDIX B

Exemplars of Reading Text Complexity and Quality, ELA K–12

Selecting Text Samples

The following text samples primarily serve to exemplify the level of complexity and quality that the *Standards* require all students in a given grade band to engage with while additionally suggesting the breadth of text types that students should encounter. The choices should serve as useful guideposts in helping educators select texts of similar **complexity**, **quality**, and **breadth** for their own classrooms. The process of text selection was guided by these criteria in the following fashion:

- *Complexity.* Appendix A describes in detail a three-part model of measuring text complexity based on qualitative and quantitative indices of inherent text difficulty balanced with educators' professional judgment. In selecting texts to serve as exemplars, the work group began by soliciting contributions from teachers, educational leaders, and researchers who have experience working with students in the grades for which the texts have been selected. These contributors were asked to propose texts that they or their colleagues have used successfully with students in a given grade band. The work group made final selections based in part on whether qualitative and quantitative measures identified by the *Standards* indicated that the proposed texts were of sufficient complexity for the grade band. For those types of texts—particularly poetry and multimedia sources—for which these measures are not as well suited, professional judgment necessarily played a greater role in selection.

- *Quality.* While it is possible to have high-complexity texts of low inherent quality, the work group solicited only texts of recognized value. From the pool of submissions gathered from outside contributors, the work group selected classic or historically significant texts as well as contemporary works of comparable literary merit, cultural significance, and/or content richness.

- *Breadth.* After identifying texts of appropriate complexity and quality, the work group applied a range of secondary criteria to ensure that the samples presented in each band represented as broad a range of sufficiently complex, high-quality texts as possible. Among the factors considered were initial publication date, authorship, and subject matter.

Copyright and Permissions

For those exemplar texts not in the public domain, the work group is seeking permission from the rights holders for limited use by the Common Core State Standards Initiative of the National Governors Association.

While we await permissions grants from the rights holders, we will make use of texts under a conservative interpretation of Fair Use, which allows limited, partial use of copyrighted text for a nonprofit, educational purpose as long as that purpose does not impair the rights holder's ability to seek a fair return for his or her work.

Please note that these texts are included solely as exemplars in support of the *Standards*. Any additional use of those texts that are not in the public domain, such as for classroom use or curriculum development, requires independent permission from the rights holders. The texts may not be copied or distributed in any way other than as part of the overall Common Core Standards Initiative document.

Organization and Excerpting

Texts are organized first by category, with narrative texts followed by drama and poetry and then the informational texts. Within each category, the texts are organized by date, usually of first publication, beginning with the oldest and ending with the most recent. In some cases, the date of any given work may be open to debate.

The excerpts given here are meant to stand in for the full work in most instances. Works that are not in the public domain may be represented by short excerpts or snippets while the work group awaits permission from the rights holders for full use.

Media Texts

Selected excerpts are accompanied by annotated links to related media texts available online at the time of the publication of this document.

Kindergarten to Grade 1 Exemplar Texts

Narratives

Little Bear by Else Holmelund Minarik, illustrated by Maurice Sendak (1957)

Are You My Mother? by P. D. Eastman (1960)

The Fire Cat by Esther Averill (1960)

Green Eggs and Ham by Dr. Seuss (1960)

Put Me in the Zoo by Robert Lopshire (1960)

Frog and Toad Together by Arnold Lobel (1971)

Owl at Home by Arnold Lobel (1975)

Henry and Mudge: The First Book of Their Adventures by Cynthia Rylant, illustrated by Suçie Stevenson (1987)

Poppleton in Winter by Cynthia Rylant, illustrated by Mark Teague (2001)

Cowgirl Kate and Cocoa by Erica Silverman, illustrated by Betsy Lewin (2005)

Poetry

"Mix a Pancake" by Christina G. Rossetti (1893)

"Singing-Time" by Rose Fyleman (1919)

"Halfway Down" by A. A. Milne (1924)

"As I Was Going to St. Ives" by Unknown, collected by Peter and Iona Opie (1951)

"Drinking Fountain" by Marchette Chute (1957)

"Poem" by Langston Hughes (1958)

"Wouldn't You?" by John Ciardi (1961)

"In the Falling Snow" by Richard Wright (1973)

"Covers" by Nikki Giovanni (1980)

"It Fell in the City" by Eve Merriam (1985)

"Celebration" by Alonzo Lopez (1993)

"Two Tree Toads" by Jon Agee (2009)

Informational Texts

A Tree Is a Plant by Clyde Robert Bulla, illustrated by Stacey Schuett (1960)

My Five Senses by Aliki (1962)

Starfish by Edith Thacher Hurd, illustrated by Robin Brickman (1962)

What Do You Do With a Tail Like This? by Steve Jenkins & Robin Page (2003)

From Seed to Pumpkin by Wendy Pfeffer, illustrated by James Graham Hale (2004)

Mouse in a Meadow by John Himmelman (2005)

Petting Zoo by Dorling Kindersley (2005)

Meet the Meerkat by Darrin Lunde, illustrated by Patricia J. Wynne (2007)

"The Forest in Spring" in National Geographic Young Explorer! April 2009 (2009)

"Our Good Earth" in National Geographic Young Explorer, April 2009 (2009)

Read-Aloud Narratives

The Wonderful Wizard of Oz by L. Frank Baum (1900)

Little House in the Big Woods by Laura Ingalls Wilder, illustrated by Garth Williams (1932)

Mr. Popper's Penguins by Richard Atwater (1938)

Finn Family Moomintroll by Tove Jansson, translated by Elizabeth Portch (1948)

A Story A Story by Gail E. Haley (1970)

The Paper Crane by Molly Bang (1985)

Read-Aloud Poetry

"The Owl and the Pussycat" by Edward Lear (1871)

"April Rain Song" by Langston Hughes (1932)

"The Fox's Foray" – Traditional rhyme in Opie / *The Oxford Nursery Rhyme Book* (1955)

Over in the Meadow by John Langstaff, illustrated by Feodor Rojankovsky (1957)

Zin! Zin! Zin! a Violin by Lloyd Moss, illustrated by Marjorie Priceman (1995)

Read-Aloud Informational Texts

The Year at Maple Hill Farm by Alice and Martin Provensen (1978)

Fire! Fire! by Gail Gibbons (1984)

Follow the Water from Brook to Ocean by Arthur Dorros (1991)

Amazing Whales! by Sarah L. Thomson (2005)

Living Sunlight: How Plants Bring the Earth to Life by Molly Bang & Penny Chisholm, illustrated by Molly Bang (2009)

Grades 2–3 Exemplar Texts

Narratives

My Father's Dragon by Ruth Stiles Gannett, illustrated by Ruth Chrisman Gannett (1948)

Crow Boy by Taro Yashima (1955)

Amos & Boris by William Steig (1971)

The Treasure by Uri Shulevitz (1978)

The Stories Julian Tells by Ann Cameron (1981)

Sarah, Plain and Tall by Patricia MacLachlan (1985)

Tops and Bottoms by Janet Stevens (1995)

The Raft by Jim LaMarche (2000)

The Lighthouse Family: The Storm by Cynthia Rylant, illustrated by Preston McDaniels (2002)

The One-Eyed Giant (Book One of Tales from the Odyssey) by Mary Pope Osborne (2002)

Poetry

"Autumn" by Emily Dickinson (1893)

"Who Has Seen the Wind" by Christina G. Rossetti (1893)

"Afternoon on a Hill" by Edna St.Vincent Millay (1917)

"Stopping by Woods on a Snowy Evening" by Robert Frost (1923)

"Something Told the Wild Geese" by Rachel Field (1934)

"Grandpa's Stories" by Langston Hughes (1958)

"A Bat Is Born" by Randall Jarrell (1964)

"Knoxville, Tennessee" by Nikki Giovanni (1968)

"Weather" by Eve Merriam (1969)

"Eating While Reading" by Gary Soto (1995)

Informational Texts

A Medieval Feast by Aliki (1983)

Maps & Globes by Jack Knowlton, pictures by Harriet Barton (1985)

Sunshine Makes the Seasons by Franklyn M. Branley (1985)

From Seed to Plant by Gail Gibbons (1991)

Throw Your Teeth on the Roof: Tooth Traditions Around the World by Selby B. Beeler, illustrated by G. Brian Karas (1998)

So You Want to Be President? By Judith St.George, illustrated by David Small (2000)

Boy, Were We Wrong About Dinosaurs by Kathleen V. Kudlinski, illustrated by S.D. Schindler (2005)

Bat Loves the Night by Nicola Davies, illustrated by Sarah Fox-Davies (2008)

Moonshot: The Flight of Apollo 11 by Brian Floca (2009)

Where Do Polar Bears Live? by Sarah L. Thomson, illustrated by Jason Chin (2010)

Read-Aloud Narratives

"How the Camel Got His Hump" in *Just So Stories* by Rudyard Kipling (1902)

The Thirteen Clocks by James Thurber (1950)

The Cricket in Times Square by George Selden, illustrated by Garth Williams (1960)

The Search for Delicious by Natalie Babbitt (1969)

Bud, Not Buddy by Christopher Paul Curtis (1999)

Read-Aloud Poetry

"The Jumblies" by Edward Lear (1871)

"The Pied Piper of Hamelin" by Robert Browning (1888)

"Your World" by Georgia Douglas Johnson (1918)

"The Song of the Jellicles" by T.S. Eliot (1939)

"Fireflies" by Paul Fleischman, illustrated by Eric Beddows (1988)

Read-Aloud Informational Texts

Lincoln: A Photobiography by Russell Freedman (1987)

A Drop of Water: A Book of Science and Wonder by Walter Wick (1997)

The Museum Book: A Guide to Strange and Wonderful Collections by Jan Mark, illustrated by Richard Holland (2007)

What the World Eats by Faith D'Aluisio, photographed by Peter Menzel (2008)

Wild Tracks! A Guide to Nature's Footprints by Jim Arnosky (2008)

Grades 4–5 Exemplar Texts

Narratives

Alice in Wonderland by Lewis Carroll (1865)

The Secret Garden by Frances Hodgson Burnett (1911)

The Black Stallion by Walter Farley (1941)

The Little Prince by Antoine de Saint-Exupery (1943)

Tuck Everlasting by Natalie Babbitt (1975)

"Zlateh the Goat" by Isaac Bashevis Singer (1984)

M. C. Higgins, the Great by Virginia Hamilton (1993)

The Birchbark House by Louise Erdrich (1999)

Bud, Not Buddy by Christopher Paul Curtis (1999)

[Also a read-aloud narrative at Grades 2–3]

Where the Mountain Meets the Moon by Grace Lin (2009)

Poetry

"The Echoing Green" from *Songs of Innocence* by William Blake (1789)

"The New Colossus" by Emma Lazarus (1883)

"Casey at the Bat" by Ernest Lawrence Thayer (1888)

"A Bird Came Down the Walk" by Emily Dickinson (1893)

"Fog" by Carl Sandburg (1916)

"Dust of Snow" by Robert Frost (1923)

"Little Red Riding Hood and the Wolf" by Roald Dahl (1982)

"They Were My People" by Grace Nichols (1988)

"Words Free As Confetti" by Pat Mora (1996)

Informational Texts

Discovering Mars by Melvin Berger (1992)

Let's Investigate Marvelously Meaningful Maps by Madelyn Wood Carlisle (1992)

Hurricanes: Earth's Mightiest Storms by Patricia Lauber (1996)

The Kid's Guide to Money by Steve Otfinoski (1996)

Toys: Amazing Stories behind Some Great Inventions by Don Wulffson (2000)

"Good Pet, Bad Pet" by Elizabeth Schleichert from *Ranger Rick* (2002)

"Ancient Mound Builders" by E. Barrie Kavash from *Cobblestone* (2003)

About Time: A First Look at Time and Clocks by Bruce Koscielniak (2004)

England the Land by Erinn Banting (2004)

A History of US by Joy Hakim (2005)

My Librarian Is a Camel by Margriet Ruurs (2005)

Horses by Seymour Simon (2006)

Quest for the Tree Kangaroo by Sy Montgomery (2006)

Volcanoes by Seymour Simon (2006)

We Are the Ship: The Story of Negro League Baseball by Kadir Nelson (2008)

"Kenya's Long Dry Season" by Nellie Gonzalez Cutler from *Time for Kids* (2009)

"Seeing Eye to Eye" by Leslie Hall from *National Geographic Explorer* (2009)

"Computer" from *Britannica Junior Encyclopedia* (2010)

"Telescopes" by Ronan, Colin A. from *The New Book of Knowledge* (2010)

"Underground Railroad" by Henrietta Buckmaster from *The New Book of Knowledge* (2010)

Grades 6–8 Exemplar Texts

Narratives

Little Women by Louisa May Alcott (1869)

The Adventures of Tom Sawyer by Mark Twain (1876)

A Wrinkle in Time by Madeline L'Engle (1962)

The Dark is Rising by Susan Cooper (1973)

Dragonwings by Laurence Yep (1975)

Roll of Thunder, Hear My Cry by Mildred Taylor (1976)

"The People Could Fly" from *The People Could Fly: American Black Folktales* by Virginia Hamilton (1985)

The Tale of the Mandarin Ducks by Katherine Paterson (1990)

"Eleven" from *Woman Hollering Creek: And Other Stories* by Sandra Cisneros (1992)

Black Ships Before Troy: The Story of the Iliad by Rosemary Sutcliff (1993)

Drama

A Midsummer Night's Dream by William Shakespeare (1596)

The Diary of Anne Frank by Frances Goodrich and Albert Hackett (1958)

Poetry

"Paul Revere's Ride" by Henry Wadsworth Longfellow (1861)

"O Captain, My Captain" by Walt Whitman (1865)

"Jabberwocky" by Lewis Carroll (1872)

"Twelfth Song of Thunder" from *The Mountain Chant: A Navajo Ceremony* Navajo tradition (1887)

"The Railway Train" by Emily Dickinson (1893)

"The Song of Wandering Aengus" by W. B. Yeats (1899)

"Chicago" from *Chicago Poems* (1914) by Carl Sandburg

"Stopping by a Wood on a Snowy Evening" by Robert Frost (1923)

"I, Too" by Langston Hughes (1925)

"The Book of Questions" by Pablo Neruda (1973) translated by William O'Daly

"Oranges" from *Black Hair* (1985) by Gary Soto

"A Poem for My Librarian, Mrs. Long" from *Acolytes* (2007) by Nikki Giovanni

Informational Texts (English Language Arts)

"Allegory of the Cave" from *The Republic* by Plato (380 BCE) translated by G.M.A. Grube

"Letter on Thomas Jefferson" by John Adams (1822)

Narrative of the Life of Frederick Douglass an American Slave by Frederick Douglass (1845)

"Gettysburg Address" by Abraham Lincoln (1863)

"Lee Surrenders to Grant" by Horace Porter (1865)

"Blood, Toil, Tears and Sweat" by Winston Churchill (1940)

Travels with Charley: In Search of America by John Steinbeck (1962)

"Address to the Nation on Civil Rights" by John F. Kennedy (1963)

I Know Why the Caged Bird Sings by Maya Angelou (1969)

"Address to Students at Moscow State University" by Ronald Reagan (1988)

Grades 9-10 Exemplar Texts

Narratives

The Odyssey by Homer (8th century B.C.E.) translated by Robert Fagles

"The Nose" by Nikolai Gogol (1836) translated by Ronald Wilks

"The Gift of the Magi" by O. Henry (1906)

The Grapes of Wrath by John Steinbeck (1939)

Fahrenheit 451 by Ray Bradbury (1953)

"I Stand Here Ironing" by Tillie Olsen (1956)

The Killer Angels by Michael Shaara (1975)

The Joy Luck Club by Amy Tan (1989)

In the Time of the Butterflies by Julia Alvarez (1994)

The Book Thief by Marcus Zusak (2005)

Drama

The Tragedy of Romeo and Juliet by William Shakespeare (1592)

The Glass Menagerie by Tennessee Williams (1944)

Rhinoceros by Eugene Ionesco (1959) translated by Derek Prouse

Master Harold…and the Boys by Athol Fugard (1982)

Poetry

"Song" by John Donne (1635)

"Ozymandias" by Percy Bysshe Shelley (1810)

"The Raven" by Edgar Allen Poe (1845)

"We Grow Accustomed to the Dark" by Emily Dickinson (1893)

"Loveliest of Trees" by A. E. Houseman (1896)

"Lift Ev'ry Voice and Sing" by James Weldon Johnson (1900)

"Domination of Black" by Wallace Stevens (1916)

"Yet Do I Marvel" by Countee Cullen (1925)

"Women" by Alice Walker (1970)

"I Am Offering This Poem to You" by Jimmy Santiago Baca (1977)

Informational Texts (English Language Arts)

"Preface to Lyrical Ballads" by William Wordsworth (1800)

"Speech to the Second Virginia Convention" by Patrick Henry (1775)

"Second Inaugural Address" by Abraham Lincoln (1865)

"State of the Union Address" by Franklin Delano Roosevelt (1941)

"I Am an American Day Address" by Learned Hand (1944)

"Remarks to the Senate in Support of a Declaration of Conscience" by Margaret Chase Smith (1950)

"Address at the March on Washington" by Martin Luther King, Jr. (1963)

"Nobel Prize Acceptance Speech" by Elie Wiesel (1986)

"A Quilt of a Country" by Anna Quindlen (2001)

Grades 11-12 Exemplar Texts

Narratives

Pride and Prejudice by Jane Austen (1813)

Jane Eyre by Charlotte Brontë (1848)

"At Home" by Anton Chekov (1887) translated by Constance Garnett

The Great Gatsby by F. Scott Fitzgerald (1925)

As I Lay Dying by William Faulkner (1930)

Their Eyes Were Watching God by Zora Neale Hurston (1937)

Black Boy by Richard Wright (1945)

The Adventures of Augie March by Saul Bellow (1949)

The Bluest Eye by Toni Morrison (1970)

Dreaming in Cuban by Cristina García (1992)

The Namesake by Jhumpa Lahiri (2003)

Drama

Macbeth by William Shakespeare (c1611)

The Importance of Being Earnest by Oscar Wilde (1895)

Death of a Salesman by Arthur Miller (1949)

A Raisin in the Sun by Lorraine Hansberry (1959)

Poetry

"A Valediction Forbidding Mourning" by John Donne (1633)

"Ode on a Grecian Urn" by John Keats (1820)

"Song of Myself" from *Leaves of Grass* by Walt Whitman (c1860)

"Because I Could Not Stop for Death" by Emily Dickinson (1890)

"Mending Wall" by Robert Frost (1914)

"Ode to My Suit" by Pablo Neruda (1954) translated by Margaret Sayers Peden

"Sestina" by Elizabeth Bishop (1983)

"The Latin Deli: An Ars Poetica" by Judith Ortiz Cofer (1988)

"Demeter's Prayer to Hades" by Rita Dove (1995)

"Man Listening to Disc" by Billy Collins (2001)

Informational Texts (English Language Arts)

The Declaration of Independence by Thomas Jefferson (1776)

The Crisis by Thomas Paine (1776)

Walden by Henry David Thoreau (1854)

"Society and Solitude" by Ralph Waldo Emerson (1857)

"The Fallacy of Success" by G.K. Chesterton (1909)

The American Language by H.L. Mencken (1938)

"Politics and the English Language" by George Orwell (1946)

"Abraham Lincoln and the Self-Made Myth" by Richard Hofstadter (1948)

"Letter from Birmingham City Jail" by Martin Luther King, Jr. (1963)

"Mother Tongue" by Amy Tan (1990)

"Take the Tortillas Out of Your Poetry" by Rudolfo Anaya (1995)

The Common Core K–12 Mathematics Standards

This document provides grade level standards for mathematics in grades K–8, and high school standards organized under the headings of the *College and Career Readiness Standards in Mathematics*. Students reaching the readiness level described in that document (adjusted in response to feedback) will be prepared for non-remedial college mathematics courses and for training programs for career-level jobs. Recognizing that most students and parents have higher aspirations, and that ready for college is not the same as ready for mathematics-intensive majors and careers, we have included in this document standards going beyond the readiness level. Most students will cover these additional standards. Students who want the option of entering STEM fields will reach the readiness level by grade 10 or 11 and take precalculus or calculus before graduating from high school. Other students will go beyond readiness through statistics to college. Other pathways can be designed and available as long as they include the readiness level. The final draft of the K–12 standards will indicate which concepts and skills are needed to reach the readiness level and which go beyond. We welcome feedback from states on where that line should be drawn.

English Language Learners in Mathematics Classrooms

English language learners (ELLs) must be held to the same high standards expected of students who are already proficient in English. However, because these students are acquiring English language proficiency and content area knowledge concurrently, some students will require additional time and all will require appropriate instructional support and aligned assessments.

ELLs are a heterogeneous group with differences in ethnic background, first language, socio-economic status, quality of prior schooling, and levels of English language proficiency. Effectively educating these students requires adjusting instruction and assessment in ways that consider these factors. For example ELLs who are literate in a first language that shares cognates with English can apply first-language vocabulary knowledge when reading in English; likewise ELLs with high levels of schooling can bring to bear conceptual knowledge developed in their first language when reading in a second language. On the other hand, ELLs with limited or interrupted schooling will need to acquire background knowledge prerequisite to educational tasks at hand. As they become acculturated to US schools, ELLs who are newcomers will need sufficiently scaffolded instruction and assessments to make sense of content delivered in a second language and display this content knowledge.

While some ELLs are economically and educationally advantaged, this is not the case for many of these students. Moreover, once in the U.S., the majority of ELLs attend high poverty schools with high percentages of other ELLs. These schools often lack the resources and capacity needed to help ELLs reach high academic standards. However, schools and districts can be assisted in providing a positive learning environment that capitalizes on the linguistic and cultural diversity of the student body.

Language proficiency is a complex construct that can reflect proficiency in multiple contexts, modes, and academic disciplines. Current measures of language proficiency may not give an accurate picture of an individual's language competence. In particular, we do not have measures or assessments for language proficiency related to competence in mathematics for different ages or mathematical topics. These two facts can confuse discussions of mathematics instruction for ELLs. In particular, because of the complexity of language proficiency and the limitations of the label "English Language Learner" as currently implemented, instructional decisions should not be made solely based on that label. However, research on language and mathematics education for this student population does provide a few clear results to guide practices for teaching ELLs mathematics:

- English learners can participate in mathematical discussions as they learn English (Moschkovich, 1999a, 2002, 2007a, 2007b, 2007d).

- Mathematics instruction for students who are learning English should draw on multiple resources and modes available in classrooms—such as objects, drawings, inscriptions, and gestures—as well as home languages and mathematical experiences outside of school.

- While mathematics instruction for ELLs should address mathematical discourse and academic language, this involves much more than vocabulary instruction.

Basic principles for improving the mathematics achievement of ELLs

Language is a resource for learning mathematics, it is not only a tool for communicating, but also a tool for thinking and reasoning mathematically. All languages (English, Spanish, Tagalog,etc.) and language varieties (different dialects, home or everyday ways of talking, vernacular, slang, etc.) provide resources for mathematical thinking, reasoning, and communicating.

Regular and active participation in the classroom—not only reading and listening but also discussing, explaining, writing, representing, and presenting—is crucial to ELLs' success in mathematics, and that ELLS can produce explanations, presentations, etc. and participate in classroom discussions *as they are learning English* (Moschkovich, 1999 and 2007).

- ELLs, like English-speaking students, require regular access to teaching practices that are most effective for improving student achievement. These practices include: a) Keeping mathematical tasks at high-cognitive demand (Henningsen & Stein, 1997; Silver & Stein, 1996); b) teachers and students attend explicitly to concepts (Hiebert & Grouws, 2007), and c) students wrestle with important mathematics (Hiebert & Grouws, 2007).
- See the evidence of ELLs' mathematical thinking, hear how ELLs use language to communicate about mathematics, understand the competence that ELLS bring to the classroom, build on this competence, and provide access to opportunities for advancing their mathematics learning.

Overall, research suggests that:

- Classroom instruction should allow bilingual students to choose the language they prefer for arithmetic computation. Language switching can be swift, highly automatic, and facilitate rather than inhibit solving word problems in the second language, as long as the student's language proficiency is sufficient for understanding the text of the word problem.
- Instruction should ensure that students understand the text of word problems before they attempt to solve them.
- Instruction should include a focus on "mathematical discourse" and "academic language" because these are important for English learners. Although it is crucial that students who are learning English have opportunities to communicate mathematically, this is not primarily a matter of learning vocabulary. Students learn to participate in mathematical reasoning, not by learning vocabulary, but by making conjectures, presenting explanations, and/or constructing arguments.
- While vocabulary instruction is important, it is not sufficient for supporting mathematical communication. Furthermore, vocabulary drill and practice are not the most effective instructional practices for learning vocabulary. Instead, research has demonstrated that vocabulary learning occurs most successfully through instructional environments that are language-rich, actively involve students in using language, require that students both understand spoken or written words and also express that understanding orally and in writing, and require students to use words in multiple ways over extended periods of time (Blachowicz, Camille, and Peter Fisher, 2000). To develop written and oral communication skills, students need to participate in negotiating meaning for mathematical situations and in mathematical practices that require output from students (Moschkovich, 2009).

References

Blachowicz, Camille, and Peter Fisher. "Vocabulary Instruction." In *Handbook of Reading Research*, Volume III, edited by Michael Kamil, Peter Mosenthal, P. David Pearson, and Rebecca Barr, pp. 503-523. Mahwah, NJ: Lawrence Erlbaum Associates, 2000.

Gándara, Patricia, and Frances Contreras. *The Latino Education Crisis: The Consequences of Failed Social Policies.* Cambridge, Ma: Harvard University Press, 2009.

Moschkovich, Judit N. "Supporting the Participation of English Language Learners in Mathematical Discussions." *For the Learning of Mathematics* 19 (March 1999): 11-19.

Moschkovich, J. N. (in press). Language, culture, and equity in secondary mathematics classrooms. To appear in F. Lester & J. Lobato (Ed.), Teaching and Learning Mathematics: Translating Research to the Secondary Classroom, Reston, VA: NCTM.

Moschkovich, Judit N. "Examining Mathematical Discourse Practices," *For the Learning of Mathematics* 27 (March 2007): 24-30.

Moschkovich, Judit N. "Using Two Languages when Learning Mathematics: How Can Research Help Us Understand Mathematics Learners Who Use Two Languages?" *Research Brief and Clip*, National Council of Teachers of Mathematics, 2009 http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf. (accessed November 25, 2009).

Moschkovich, J.N. (2007) Bilingual Mathematics Learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir and P. Cobb (Eds.), *Diversity, Equity, and Access to Mathematical Ideas*. New York: Teachers College Press, 89-104.

Schleppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly,* 23:139-159.

Access for Students with Disabilities

The Common Core Standards articulate rigorous expectations in the areas of mathematics, reading, writing, and speaking and listening in order to prepare students to be college- and career-ready. These standards identify the knowledge and skills students must acquire in order to be successful. Research shows that students with disabilities are capable of high levels of learning and should not be limited by low expectations and watered down curriculum. It is imperative that these highly capable students—regardless of their disability—are held to the same expectations articulated in the Core Standards as other students.[1]

However, *how* these high standards are taught is of the utmost importance in reaching students with special needs. When learning the knowledge and skills represented in the Core Standards, students with disabilities may need accommodations or—in exceptional cases—modified goals, incorporated in an individualized education program (IEP),[2] to help them access information or demonstrate their knowledge. Students might be precluded from reaching particular standards given the nature of the standard itself. In instances when a standard asks students to perform actions they are physically incapable of, students will need to be presented with alternative options to demonstrate similar knowledge and skills within the range of their abilities. Accommodations based on individual needs allow students of all disability levels to learn within the framework of the Core.

Meeting English Language Arts (ELA) Standards

Reading, writing, speaking, and listening standards often require accommodations for students with disabilities. In the case of students who are deaf, a standard that calls for "listening" should be interpreted to include reading sign language. In a similar vein, "speaking" as it occurs in standards for certain students with speech impairments should be read broadly to include "communication" or "self-expression." Students who are blind or have low vision should be able to read via Braille, screen reader technology, or other assistive technology to demonstrate their comprehension

[1] Research suggests that the vast majority of the population of students with intellectual impairments *can* achieve proficiency when they receive high quality instruction in the grade-level content and appropriate accommodations.
[2] According to the Individuals with Disabilities Act (IDEA), an IEP includes appropriate accommodations that are necessary to measure the individual achievement and functional performance of a child.

skills. "Writing" should not preclude the use of a scribe, computer, or speech-to-text technology for students with disabilities that interfere with putting pen to paper. In the case of students with intellectual impairments—less than 2 percent of the total population of all students and less than 20 percent of students with disabilities—accommodations should allow them to demonstrate their knowledge and skills through alternative modes like text to speech software or reading aloud. For these students, writing may involve the use of pictures to assist in illustrating plot or argument, or offering them the opportunity to "choose words and phrases" by selecting from options rather than generating direct answers. With appropriate accommodations and support, students with all levels of disabilities can participate in the general education curriculum and achieve grade-level proficiency with regard to the ELA content and skills articulated in the Core.

Meeting Mathematics Standards

In curriculum for students with disabilities, ELA skills often take precedence over mathematics understanding. However, most of these students can master mathematical concepts with accommodations in instructional delivery and the use of specialized technology, including computers and calculators. For example, students with visual disabilities might require enhanced verbal descriptions from teachers and the use of large-print to demonstrate subsequent knowledge. Students who are deaf might require visual aids such as charts, diagrams, and mental images and increased reliance on computers and calculators. Manipulatives can enable students with intellectual impairments to grasp abstract concepts and continue learning. Evidence suggests that students with disabilities, even those with full intellectual abilities, tend to lag behind their peers in mathematics achievement; strong curriculum that gives equal priority to mathematics and ELA will help these students succeed.

In short, while the standards remain and retain high expectations of students, they may need to be translated and occasionally modified to appropriately apply to students with disabilities, including all levels of intellectual impairment. Every student deserves to be treated with respect, and every student deserves an outstanding education. Promoting a culture of high expectations for all students is a fundamental goal of the Core Standards. Reaching students with disabilities requires broadening our understanding of what the standards say and being ready to make appropriate accommodations and/or modifications to meet individual students' needs.

How to read this document

The K–8 standards are organized by grade level. Within each grade level there are several headings, each one the title of a single progression having significant presence in the grade in question. Under each of these progression headings, there appear **core standards**, divided into standards describing concepts students should understand and standards describing skills students should acquire. A typical progression spans a number of grades, but does not span all of K-8. The progressions and their grade spans are listed at the end of the document.

The high school standards are not organized by grade level or by course, but rather are organized under headings of the *College and Career Ready Standards for Mathematics*: Expressions, Equations, Functions, Coordinates, Modeling, Statistics, Probability, and Geometry.[3] Subheadings under each heading refer either to mathematical practices or to principle topics, and core standards are listed under each subheading, as in the K-8 standards. The subheadings are not necessarily curricular units, but rather can describe concepts and skills that are revisited throughout a student's high school career. This design necessitates a future effort to develop course sequences (either traditional or integrated).

[3] Number and Quantity are not included, since they are principally the domain of K–8. In response to feedback, the headings have been reordered and Shape has been renamed to Geometry.

Mathematical Practice[4]

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement.[a] These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures.[b] Taken together with the Standards for Mathematical Content, they support productive entry into college courses or career pathways.

Core Standards · Students can and do:

1 Attend to precision.

Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies noting the assumptions made. They clarify definitions. They state the meaning of the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. Rather than saying, "let v be speed and let t be time," they would say "let v be the speed in meters per second and let t be the elapsed time in seconds from a given starting time." They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits indicate unwarranted precision.

2 Construct viable arguments.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

3 Make sense of complex problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

4 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern. For example, in $x^2 + 5x + 6$ they can see the 5 as $2 + 3$ and the 6 as 2×3. They recognize the significance of an existing line in a geometric figure and can add an auxiliary line to make the solution of a problem clear. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects. For example, by seeing $5 - 3(x$

[4] Slated for review and editing, based on feedback to the College and Career Readiness Standards, and in order to apply more naturally to elementary school as well.

$- y)^2$ as 5 minus a positive number times a square, they see that it cannot be more than 5 for any real numbers x and y.[b]

5 Look for and express regularity in repeated reasoning.

Mathematically proficient students pay attention to repeated calculations as they carry them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, they might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel in the expansions of $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ leads to the general formula for the sum of a geometric series. As they work through the solution to a problem, proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.[b]

6 Reason quantitatively.

Quantitative reasoning is a way of thinking by which one reasons with quantities and about relations among quantities. It entails habits of creating a coherent image of the problem at hand; considering the units involved; continually attending to the meaning of quantities, not just how to compute them; and having multiple images of a concept and being flexible in transitioning among them. In problems dealing with quantitative relationships, students exercise two inseparable abilities: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referential meanings for the symbols involved in the manipulation.

7 Make strategic decisions about the use of technological tools.

Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, ruler, protractor, graphing calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding and estimation strategically, attending to levels of precision, to ensure appropriate levels of approximation and to detect possible errors. They are able to use these tools to explore and deepen their understanding of concepts.

Mathematics: Kindergarten[5]

Developing Coherent Understanding

[Temporarily removed for editing.]

Counting and Cardinality

Core Standards · Students understand that:

1. The number words have a standard order.
2. In counting, each object receives one and only one number word.
3. The last number word tells the number of objects.
4. Numbers said later in the count refer to larger quantities.
5. Counting on 1 more is the same as adding 1. That is, one more than a number is the next number in the count.

Core Standards · Students can and do:

a. Count by ones from 1 to 100; count by tens to 100.[6]
b. Count forward from a given number within the known sequence (instead of always counting forward from 1).[7]
c. See collections of up to 10 objects as being composed of subgroups.
d. Count to answer "how many?" questions with up to 10 things in various arrangements (e.g., array, circular, scattered), or up to 25 things if in a row.
e. Write numerals from 1 to at least 30.

Base Ten Computation

Core Standards · Students understand that:

1. Ten ones make a tens unit (ten things can be thought of as bundled into a single unit).
2. Decade words refer to groups of tens units. For example, thirty refers to a group of three tens units.
3. A teen number[8] is a ten and some ones. The number 10 can be thought of as a ten and no ones.
4. Any teen number is larger than any single digit number. Teen numbers are ordered according to their ones digits.
5. A two-digit number is some tens and some ones. For example, 29 is two tens and nine ones.

Core Standards · Students can and do:

a. Make 10 with each number from 1 to 9 (i.e., know the number that makes 10 with the given number).
b. Show each teen number as a ten and some ones.

Early Relations and Operations

Core Standards · Students understand that:

[5] Some material is used verbatim from National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Committee on Early Childhood Mathematics, Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.
[6] To "count" here means only to say the number words, not to determine how many objects are in a collection.
[7] To "count" here means only to say the number words, not to determine how many objects are in a collection.
[8] Glossary: Teen number. A whole number that is greater than or equal to 11 and less than or equal to 19.

1. Adding is putting two groups together or putting some more with a group, and subtracting is taking some from a group.
2. Addition and subtraction can be represented with physical or mental objects (including fingers), pictures, drawings, sounds (e.g. number words), motions, or equations.
3. Adding can be recorded by an expression, as when "three more than six" is recorded as 6 + 3, or by an equation that also shows the answer (6 + 3 = 9). Likewise, subtracting can be recorded by an expression, as when "how much more than 9 is 5" is recorded as 9 – 5, or by an equation that also shows the answer (9 – 5 = 4).
4. Breaking apart a group can be recorded in an equation such as 8 = 5 + 3. Breaking apart a group in more than one way can be recorded in an equation such as 7 + 6 = 10 + 3.
5. In all equations, the equals sign indicates that the values on either side are the same.

Core Standards · Students can and do:

a. Use matching and counting strategies to decide whether one set is more than, less than, or equal to another set in number of objects (less than or equal to 10).
b. Compare and order numbers less than or equal to 10.
c. Use concrete objects to determine the answer to addition and subtraction word problems and additions and subtractions with totals less than or equal to 10.
d. Experience enough problem situations so that additions to five and the corresponding subtractions and some additions and subtractions within ten become well known.

Quantity and Measurement

Core Standards · Students understand that:

1. Things have attributes—such as length, weight, capacity, loudness, softness, and so on. A single thing might have several attributes of interest (as when we focus on a child's height and gender).

Core Standards · Students can and do:

a. Directly compare two objects to see which one has "more of" a shared attribute.
b. Rank three objects by a shared attribute (especially length), and use transitivity[9] to compare two objects indirectly.
c. ♦ Classify objects or people into predetermined categories, and count the numbers in each category. List the categories and counts in order by count. (Each category count less than or equal to 10.)[10]

Shapes

Core Standards · Students understand that:

1. Names refer to shapes regardless of orientation or overall size.[11]

Core Standards · Students can and do:

a. Study a range of 2D and 3D shapes, in different sizes and orientations, and discuss their properties, similarities, and differences using informal language.

[9] Glossary: Transitive property of measurement order: If one object is bigger than a second, and the second object is bigger than a third object, then the first object is bigger than the third object.
[10] The symbol ♦ indicates material in data analysis and statistics that appears under another progression heading in order to make an important connection.
[11] For example, a square rotated to form a "diamond" is still a square, even though it is rotated. Students at this grade might need to physically rotate a shape until it is "level" before they can correctly name it.

b. Move shapes using translations, reflections and rotations.[12]

[12] This is not meant to be assessed by showing students a picture of a shape and asking them to draw or select a translated, reflected or rotated version of it.

Mathematics: First Grade[13]

Developing Coherent Understanding

[Temporarily removed for editing.]

Early Relations and Operations

Core Standards · Students understand that:

1. Counting on is an efficient method of counting all, in which the initial count of the first addend is omitted.
2. Addition and subtraction apply to situations of joining, separating, part-part-whole, and comparing quantities to one another.[14] These situations can be represented by addition and subtraction equations such as 7 + 5 = 12, 10 = 5 + 5, and so on.
3. Addition and subtraction are inverse operations; that is 10 – 8 can be found by thinking 8 + 2 = 10.
4. When any two of the numbers in an addition or subtraction equation are known, the unknown number can be found.
5. One-to-one dealing of objects in a collection (e.g., "One for you, one for me, one for him, …") creates fair shares.

Core Standards · Students can and do:

a. Use counting on strategies or decomposing strategies for additions and subtractions within 20.
b. Solve addition problems containing three addends.
c. Use objects, pictures and story contexts to explain what happens when the order of addends in a sum is changed, when 0 is added to a number, and when one addend in a sum is increased by 1 and another decreased by 1.
d. Experience enough problem situations so that many or all sums and differences within 20 become well known.
e. Use drawings and equations to represent and solve word problems involving addition and subtraction.[15]
f. ♦ Organize, represent and interpret data with several categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another
g. Create *n* fair shares from a collection of objects. Identify the size of one share, and recognize the original collection as *n* copies of a single share.

Quantity and Measurement

Core Standards · Students understand that:

1. Lengths can be added by placing long objects, rods, or unit cubes end to end in a straight line. The total length is the same in whatever order the rods are placed.

[13] Some material is used verbatim from National Research Council. (2009, op. cit.)
[14] In join and separate problems, there is change over time. In part-part-whole problems, two quantities make up a whole in a static situation. Compare problems involve two quantities and the difference between them. Compare problems add specificity to the notions of greater than and less than.
[15] Include join, separate, part-part-whole, and compare problems, with unknowns in all positions. Represent these situations with equations that use a small square or a ? for the unknown.

2. Lengths can be compared by placing rods side by side, with one end lined up. The difference in length is how far the longer extends beyond the end of the shorter.
3. Lengths are measured (assigned numerical values) by comparing them to other lengths—that is, by using another object as a length unit. The length of an object can be expressed numerically by counting the number of length units that span it with no gaps or overlaps.
4. When an object or figure is decomposed into several pieces, the length of the whole can be found by placing the pieces end to end in any order.
5. A sum of two whole numbers represents a total length; a difference of two whole numbers represents a difference in length.
6. Durations of time are measured by comparing them to other durations of time, such as the earth's rotation period, or the time a minute hand takes to complete a circle around a clock face.

Core Standards · Students can and do:

a. Using an object as a length unit, measure, compare and estimate length.[16]
b. Using an object as a length unit, determine total length by adding lengths of two parts.[17] Compare lengths using addition and subtraction.
c. Decompose circles and rectangles into 2 and 4 equal parts. Describe the parts using the words "halves" and "quarters," and using the phrases "half of" and "quarter of." Describe the wholes as twice or four times as large as the parts.
d. Tell time in hours from clocks; subtract to find whole-hour durations on a clock (within AM or within PM).

Base Ten Computation

Core Standards · Students understand that:

1. In comparing two-digit numbers, the number with more tens units is larger; if the number of tens units is the same in each, the number of ones units decides.
2. In adding or subtracting 2-digit numbers, one adds or subtract like units (tens units and tens units, or ones units and ones units).

Core Standards · Students can and do:

a. Count to 100 or beyond, switching appropriately to the new decade after a 9 has been said in the ones place.
b. Compare and order numbers to 100 based on meanings of the tens and ones places.
c. Easily write numerals to 20; write numerals to 100.
d. Use break-apart and make-a-ten strategies to add and subtract with teen totals as in $7 + 6 = 10 + 3$ and $17 - 9 = 17 - 7 - 2$.
e. Find 10 more or 10 less than a number without having to count.
f. Add one-digit numbers to two-digit numbers, and add multiples of 10 to one-digit and two-digit numbers.
g. Represent addition of two-digit numbers using 10-rods and unit cubes,[18] including rearranging rods and cubes to show regrouping when needed.
h. Add two-digit numbers to two-digit numbers using strategies based on place value, Properties of Arithmetic, or the inverse relationship between addition and subtraction.

Shapes

[16] Select and iterate units, partition into equal parts, and compare lengths indirectly by using a reference length.
[17] Restrict to whole-unit lengths.
[18] Any concrete model that can show individual units and ten connected units will do.

Core Standards · Students understand that:

1. Several shapes can be joined together to form a larger shape. A single shape can also be visualized as a collection of smaller shapes joined together.
2. Decomposing larger shapes into equal-sized parts creates fair shares.
3. When an identical figure is decomposed into more fair shares, the shares are smaller than in the first instance.

Core Standards · Students can and do:

a. Form different 2D figures with cutouts of rectangles, squares, triangles, semicircles, and quarter-circles.[19]
b. Form different 3D figures with concrete models of cubes, rectangular prisms, cones, and cylinders.[20]
c. Decompose 2D shapes into rectangles, squares, triangles, semicircles, and quarter-circles, including decomposing into fair shares.

[19] From Singapore Primary 2
[20] From Singapore Primary 2

Mathematics: Second Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Operations and the Problems They Solve

Core Standards · Students understand that:

1. Addition and subtraction apply to situations of joining, separating, part-part-whole, and comparing quantities to one another.[21] These situations can be represented by addition and subtraction equations such as 17 + 5 = 22, 36 = 56 – 26, and so on.
2. Addition and subtraction are inverse operations; that is 100 – 98 can be found by thinking 98 + 2 = 100.
3. Numbers can be added and subtracted only when they refer to the same underlying unit.

Core Standards · Students can and do:

a. Use representations (objects, pictures, story contexts) to describe and justify properties of addition and subtraction.[22]
b. Produce full sets of related equations for addition and subtraction, as in the set 5 + 3 = 8, 3 + 5 = 8, 8 = 5 + 3, 8 = 3 + 5, 8 – 5 = 3, 8 – 3 = 5, 3 = 8 – 5, 5 = 8 – 3.
c. Solve up to two-step addition/subtraction word problems with whole numbers and whole number quantities within 100.[23]

Base Ten Computation

Core Standards · Students understand that:

1. A three-digit number is made up of hundreds, tens and ones units. Digits in each place are worth ten times as much as digits in the place to the right.
2. Comparison of numbers is decided by the leftmost digit, with subsequent digits breaking ties.
3. Three-digit numbers can be expanded into sums of hundreds, tens and ones units. In adding or subtracting, one adds or subtracts the units of each size; regrouping might be needed to write a total in standard form if there are too many of a unit, or to get enough of a unit to subtract from it.
4. The scheme for regrouping is the same at each place, because each unit is composed of ten of the smaller unit.

Core Standards · Students can and do:

a. Compare and order numbers to 1,000.

[21] In join and separate problems, there is change over time. In part-part-whole problems, two quantities make up a whole in a static situation. Compare problems involve two quantities and the difference between them. Compare problems add specificity to the notions of greater than and less than.
[22] Include properties such as that the sum is the same when multiple addends are added in a different order; if adding two numbers gives a certain sum, then subtracting one of the addends from the sum results in the other addend; that if more is subtracted from a number, the difference is decreased and if less is subtracted the difference is increased; that in an addition problem, each addend can be taken apart and the parts can be recombined in any order without changing the sum.
[23] Include join, separate, part-part-whole, and compare problems, with unknowns in all positions. Represent these situations with equations that use a small square or a ? for the unknown.

b. Given a three-digit number, quickly find 10 more or 10 less than the number, and quickly find 100 more or 100 less than the number.
c. Rapidly add and subtract within 20.[24]
d. Add and subtract three-digit numbers to three-digit numbers using strategies based on place value, Properties of Arithmetic, or the inverse relationship between addition and subtraction.
e. Add and subtract three-digit numbers using an algorithm[25] based on place value and regrouping, such as the standard algorithm.
f. Explain why addition and subtraction strategies and algorithms work, using place value and the Properties of Arithmetic (including explanations supported by drawings or objects).

Quantity and Measurement

Core Standards · Students understand that:

1. 1 inch, 1 foot, 1 centimeter and 1 meter are conventionally defined lengths that allow standardized length measurements.
2. When measuring a length, if a smaller unit is chosen, more units must be iterated to measure the length in those units. But the length of an object itself does not depend on the choice of unit.
3. Units can be decomposed into smaller units, e.g. a foot contains 12 inches and a meter contains 100 centimeters. A small number of long units might form a greater total length than a large number of small units.
4. Sharing a circle or rectangle fairly among 2-6 shares creates equal parts, each of which is a single unit. Copying one unit by the number of pieces measures the whole in terms of the units.
5. A half, a third, or a quarter of a given rectangle encloses the same amount of space regardless of its shape.

Core Standards · Students can and do:

a. Measure, compare and estimate whole-unit lengths in units of inches, feet centimeters and meters.
b. Construct a number line with an origin (0) and a unit (1), marking off whole numbers one unit distance apart. Use a number line to represent sums and differences; determine lengths of intervals on the number line.
c. Decompose circles and rectangles into 2-6 equal parts. Describe the parts using the words "halves," "thirds," "half of," "a third of," etc. Describe the wholes as 2-6 times as large as the parts.
d. Construct a number line to 100 using tens-unit lengths, showing ones-unit lengths within a decade of interest. Explain regrouping by composing and decomposing concrete lengths.
e. ♦ Draw a bar graph (with single-unit scale) to represent a data set with several categories. Solve simple part-part-whole and compare problems using information presented in a bar graph.[26]
f. ♦ Identify correspondences in different representations of a data set with several categories.
g. Solve word problems involving dollar bills, quarters, dimes, nickels and pennies.

Shapes

Core Standards · Students understand that:

[24] Acceptable strategies include: mental strategies such as making a ten, use of fingers to assist in rapid counting-on, and producing sums or differences from memory.
[25] Glossary: Algorithm. A step by step routine that always gives some answer, rather than ever giving no answer; that always gives the right answer, and never gives a wrong answer; that can always be completed in a finite number of steps, rather than in an infinite number of steps; and that applies to all problems of a given type (e.g., adding any two multidigit whole numbers, or bisecting any angle). Cf. Wikipedia's "effective procedure," from which this definition is adapted.
[26] For part-part-whole problems, only sum-unknown problems are required to meet this standard. For compare problems, only difference-unknown problems are required to meet the standard.

1. A given category of shapes (e.g., triangles) can be divided into subcategories (e.g., isosceles triangles) on the basis of special properties. Conversely, different classes of shapes (e.g., squares and rectangles) can be united into a larger category (e.g., quadrilaterals) on the basis of shared properties.

Core Standards · Students can and do:

a. Draw and identify equilateral triangles, isosceles triangles,[27] squares and rectangles.[28]
b. Recognize squares and rectangles as examples of quadrilaterals; draw examples of quadrilaterals that are neither squares nor rectangles.
c. Draw and identify radii and diameters of a circle.
d. Recognize objects that resemble spheres, cylinders and rectangular prisms.

[27] Students at this grade need not understand that equilateral triangles are isosceles.
[28] Students at this grade need not understand that squares are rectangles.

Mathematics: Third Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Operations and the Problems They Solve

Core Standards · Students understand that:

1. Multiplication and division apply to situations of equal grouping, fair sharing, measuring, and comparing ("times as much").
   - An equation of the form $a \times b = n$ applies to a situation in which a groups of b things each make n things in all, or in which a copies of a continuous quantity of size b form a continuous quantity of size n. (See table for examples.)
   - An equation of the form $n \div a = b$ tells how many things, b, are in each group when n things are divided equally into a groups, or tells how large a quantity b results when a continuous quantity of size n is shared fairly into a shares. (See table for examples.)
   - An equation of the form $n \div b = a$ tells how many groups, a, result when n things are divided into equal groups of b things each, or tells how many fair shares, a, a quantity of size n yields when each share has size b. (See table for examples.)
   - Two quantities can be compared by multiplication or division. An equation of the form $a \times b = n$ means n is a times as much as b and b times as much as a.
2. Multiplication is commutative: The total number of things in a groups of b things each is the same as the total number of things in b groups of a things each, that is, $a \times b = b \times a$. Likewise, a copies of a continuous quantity of size b are equal in size to b copies of a continuous quantity of size a.
3. The area of a rectangle with whole number side lengths can be calculated by multiplying because the rectangle can be decomposed into equal rows (or columns) of unit squares.
4. Multiplication and division are inverse operations; that is $35 \div 7$ can be found by thinking $5 \times 7 = 35$. When any two of the numbers in a multiplication or division equation are known, the unknown number can be found.

	$3 \times 6 = 18$	$18 \div 3 = 6$	$18 \div 6 = 3$
Collections	3 rows of apples with 6 apples in each row are 18 apples.	If 18 apples are arranged into 3 equal rows, each row will have 6 apples in it.	If 18 apples are arranged into equal rows of 3 apples, there will be 6 rows.
Continuous Quantities	If you have enough ribbon to make 6 bows, then 3 times as much ribbon will make 18 bows.	If you have enough ribbon to make 18 bows and share the ribbon fairly among 3 kids, then each kid has enough ribbon to make 6 bows.	If each kid wants to make 6 bows and there's enough ribbon to make 18 bows, then 3 kids can make bows.

Core Standards · Students can and do:

a. Use representations (objects, pictures, story contexts) to describe and justify properties of multiplication and division.[29]
b. Solve simple multiplication and division word problems involving equal groups, length and area.
c. Solve up to two-step word problems involving the four operations with whole numbers and whole number quantities. (Whole number quotients only)
d. Solve multiplicative comparison problems with whole numbers (problems involving the notion of "times as much").
e. ♦ Draw a scaled bar graph to represent a data set with several categories. Solve "how many more"/"how many less" problems (two-step problems) using information presented in scaled bar graphs.[30]

Base Ten Computation

Core Standards · Students understand that:

1. Patterns in the multiplication table can be explained by the Properties of Arithmetic. For example, the distributive property explains why, for any row, the entries in the 7 column are the sums of the entries in the 5 and 2 columns.
2. The Properties of Arithmetic can be used to derive new multiplications and divisions from known ones.

Core Standards · Students can and do:

a. Explain strategies for multiplying and dividing that use the Properties of Arithmetic and properties of the base ten system.
b. Rapidly multiply and divide within 100.[31]
c. Produce full sets of fact families for multiplication and division, as in the set $6 \times 7 = 42$, $7 \times 6 = 42$, $42 = 7 \times 6$, $42 = 6 \times 7$, $42 \div 7 = 6$, $42 \div 6 = 7$, $6 = 42 \div 7$, $7 = 42 \div 6$.
d. Find the factor pairs for a given number, as in the factor pairs for the number 42: {42, 1}, {21, 2}, {14, 3}, {7, 6}.

Fractions

Core Standards · Students understand that:

1. When a whole, 1, is divided into b equal parts, the size of the parts is written $1/b$. To show $1/b$ of something, divide the thing into b equal parts.
2. For a whole number a and a positive whole number b, a/b is defined as a copies of $1/b$.[32] This can be thought of as the sum $1/b + 1/b + \ldots + 1/b$ (with a summands).
3. Whole numbers can be written as fractions, as in $b/b = 1$, $n/1 = n$, and cases such as $(4 \times 7)/4 = 7$.
4. Fractions are numbers and can be seen as lengths on a number line.[33]

[29] Include properties such as that the product is the same when the order of the factors is changed; that multiplication problems involving 1-digit numbers can be solved by breaking one factor apart additively and multiplying each part by the other factor; and that multiplying a quantity by a number, then dividing by the same number, leaves the original quantity unchanged.

[30] Include single-unit scales and multiple-unit scales. For multiple-unit scales, all counts should be evenly divisible by the scale factor. No count should represent more than ten of the scale unit, and no scale unit should represent more than ten counts.
[31] A variety of mental strategies are acceptable, including derived fact strategies and producing products or quotients from memory.
[32] This includes fractions greater than 1. For example, 17/5 is 17 copies of 1/5 .
[33] For example, 17/5 is 17 copies of the subinterval 1/5 laid end to end.

5. Two fractions are equal when they represent the same portion of a whole, or when they have the same length on a number line. One fraction is greater than another when it represents a greater portion of the whole than the other, or lies to the right of the other on the number line.
6. Given two unit fractions, the fraction with the larger denominator is smaller, because dividing a whole into a larger number of parts leads to smaller parts.
7. Fractions with the same denominator can be added or subtracted by adding or subtracting the units indicated by the unit fraction. For example, $2/3 + 4/3$ is 2 copies of $1/3$ plus 4 copies of $1/3$, or 6 copies of $1/3$ in all, that is $6/3$.
8. The decimal 0.1 denotes the fraction $1/10$, 0.2 denotes $2/10$, and so on through 0.9, which denotes $9/10$.

Core Standards · Students can and do:

a. Use fractions to describe quantities and parts of wholes.
b. Compare and order fractions with equal numerators or equal denominators, including in contextual situations, using the fractions themselves, bar strip drawings, number line representations, and area models.
c. Reason about fractions to establish equivalences between fractions with unlike denominators 2, 3, 4 and 6 (e.g. $1/2 = 2/4$, $4/6 = 2/3$).
d. Add and subtract fractions with like denominators.
e. Solve word problems that involve adding, subtracting, ordering and comparing fractions.
f. Represent fractions of the form $a/10$ in decimal notation; compare and order to tenths in decimal notation.

Quantity and Measurement

Core Standards · Students understand that:

1. A unit of measure can be partitioned into equal-sized parts, whose sizes can be represented as fractions of the unit.
2. The area of a closed plane figure is a measure of how much space it encloses. A square with side length 1 unit is said to enclose "one square unit" of area.
3. The area of a closed plane figure can be measured (expressed numerically) by the number of square units that fit inside it with no gaps or overlaps.
4. Area is a model for multiplication because tiling a rectangle with unit squares shows that a rectangle a units long by b units wide encloses an area of $a \times b$ square units.

Core Standards · Students can and do:

a. Measure lengths using rulers marked with halves and fourths of inches. Make a dot plot to show repeated measurements.
b. Convert compound units to a smaller or a larger unit, and solve problems involving mixed units (feet and inches, yards and feet).
c. Using customary units, demonstrate and justify correct processes for measuring, comparing, and estimating length, mass, capacity, and durations of time, including unit selection, partitioning and iterating units, and transitivity.
d. Compute perimeters of polygons by adding given side lengths, and find an unknown length in a polygon given the perimeter and all other side lengths. Represent these problems with equations involving a symbol for the unknown quantity.
e. Determine and compare areas by counting square units (improvised units, cm^2, m^2, in^2, ft^2).
f. Compute elapsed time and solve problems involving elapsed time (to the nearest minute).

Mathematics: Fourth Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Operations and the Problems They Solve

Core Standards · Students understand that:

1. Quantities in a problem might be described with whole numbers, fractions or decimals; the operations used to solve the problem depend on the relationships between the quantities whatever numbers are involved.
2. The distributive property (of multiplication over addition) relates addition and multiplication. The distributive property can be shown numerically and visually, using arrays and area models.

Core Standards · Students can and do:

a. Solve multistep word problems involving the four operations with whole numbers.[34]
b. Estimate answers to computations and compute mentally to assess reasonableness of results.
c. Solve problems that involve comparing, ordering, adding and subtracting fractions with like denominators. Compare fractions to benchmark fractions.
d. Solve problems that involve comparing and ordering decimal numbers to hundredths. Compare decimals to benchmark decimals.
e. ♦Make a table from given data, ask and answer questions about data in a table, solve multi-step problems using information presented in tables, and find patterns in tables.[35]

Fractions

Core Standards · Students understand that:

1. The fraction a/b can be written as $a \times 1/b$ because a/b is a copies of $1/b$.
2. When a identical things are divided into b equal parts, each of a things contributes $1/b$. So $a \div b = a/b$.[36]
3. A fraction can be multiplied by a whole number as $n \times a/b = n \times a/b$. For example, $3 \times 2/5$ can be seen as 3 groups of 2 unit fractions $1/5$.[37]
4. A decimal of two digits stands for a sum of fractions whose denominators are 10 and 100. For example, 0.34 stands for $3/10 + 4/100$.

Core Standards · Students can and do:

a. Reason about fractions to establish equivalences between related fractions[38] (e.g. $3/10 = 30/100$, $9/12 = 3/4$).

[34] Use the properties of multiplication (commutative, associative, identity) or the inverse relationship between multiplication and division (multiplying a number by b then dividing by b, and vice versa, leaves the number unchanged) to make sense of single digit multiplication and division situations and solve problems.
[35] Include tables with data from proportional relationships.
[36] This definition agrees with previous understandings of division in cases like $28 \div 7$ (i.e., when a is a multiple of b), but also gives meaning to quotients such as $3 \div 4$ or $7 \div 2$.
[37] Using the Properties of Arithmetic, $n \times a/b = n \times (a \times 1/b) = (n \times a) \times 1/b = (n \times a)/b$.

b. Add and subtract related fractions in simple cases within one whole (e.g. $1/2 + 1/4$, $3/10 + 4/100$).
c. Solve word problems posed with whole numbers that have fractional answers.
d. Represent multiplication of whole numbers by fractions and fractions by whole numbers, using length and area models.
e. Solve word problems involving multiplying fractions by whole numbers and multiplying whole numbers by fractions.[39]
f. Use decimals to hundredths to describe quantities and parts of wholes, compare and order decimals to hundredths, and write fractions of the form $a/10$ or $a/100$ in decimal notation.
g. Round decimals (to hundredths) to the nearest whole number.
h. Solve addition and subtraction story problems involving fractions with related denominators (situations familiar from whole number work).

Base Ten Computation

Core Standards · Students understand that:

1. The product of a one-digit number times a multidigit number is the sum of the products of the one-digit number times each place value component. This is an instance of the distributive property.
2. Multi-digit multiplication algorithms can be derived and explained by decomposing numbers into their place value components and applying the distributive property.
3. Digits in each place are worth ten times as much as digits in the place to the right and a tenth as much as digits to the left; comparison of numbers is decided by the leftmost digit, with subsequent digits breaking ties.
4. Given whole numbers a and b, find whole numbers Q and R so that $a = Q \times b + R$. For example, given 325 and 7, express 325 in the form $325 = 46 \times 7 + 3$.

Core Standards · Students can and do:

a. Demonstrate place value understanding for whole numbers to 1,000,000 and compare numbers within this range.
b. Round whole numbers to the nearest 10 or 100 and use rounding to estimate computations.
c. Multiply single place numbers (to 9000) by single digit numbers.[40]
d. Multiply two-, three- and four-digit numbers by single-digit whole numbers, and multiply two-digit numbers by two-digit numbers, using strategies based on place value, Properties of Arithmetic, or the inverse relationship between multiplication and division.
e. Multiply two-digit numbers by two-digit numbers using an algorithm based on place value and regrouping, such as the standard algorithm.
f. Divide two-, three- and four-digit numbers by single-digit numbers, with or without remainder. In the case of remainders, express results in the form of an equation, as in $325 = 46 \times 7 + 3$.
g. Explain why multiplication and division strategies and algorithms work, using place value and the Properties of Arithmetic (including explanations supported by drawings or objects).

Quantity and Measurement

Core Standards · Students understand that:

[38] Glossary: Related fractions. Two fractions are *related* if one denominator is a factor of the other. (See Ginsburg, Leinwand and Decker (2009), *Informing Grades 1-6 Mathematics Standards Development: What Can Be Learned from High-Performing Hong Kong, Korea, and Singapore?*, Table A1, p. A-5, grades 3 and 4.)
[39] Include sharing multiple continuous wholes a fairly among b people, naming an individual share as a/b. For example 5 meters of pink ribbon shared among 3 people results in $5/3$ meters each.
[40] Glossary: Single-place number. The numbers that result when a whole number between 1 and 9 (inclusive) is multiplied by the numbers 10, 100, 1000, etc.

1. Area is additive: If a figure is decomposed into several pieces, then the area of the whole figure can be found by adding the areas of the pieces (expressed in common units).
2. An angle is two rays with a common endpoint, and is measured by the relative amount of a circle that you trace when turning from one ray to the other.
3. A one-degree angle turns through 1/360 of a circle, where the circle is centered at the origin of the rays; the measure of an angle is the number of one-degree angle turned with no gaps or overlaps.

Core Standards · Students can and do:

a. Apply the formula for area of squares and rectangles. Measure and compute whole-square-unit areas of objects and geometric figures decomposable into rectangles.[41]
b. ♦ Make a dot plot to show repeated measurements in common fractions of a unit ($1/2$, $1/4$, $1/8$). Solve problems involving addition and subtraction of fractions by using information presented in dot plots (e.g., finding the difference in length between the longest and shortest specimens in an insect collection).
c. Draw scales (number line representations) of problem situations involving length, height and distance including fractional units or decimal numbers.
d. Find one dimension of a rectangle given the other dimension and its area or perimeter; find the length of one side of a square given its area or perimeter. Represent these problems with equations involving a symbol for the unknown quantity.
e. Measure angles in whole-number degrees using a protractor; sketch angles of specified measure. Find the measure of a missing part of an angle, given the measure of the angle and the measure of a part of it; represent these problems with equations involving a symbol for the unknown quantity.

Shapes

Core Standards · Students understand that:

1. Shapes can be analyzed and classified using concepts of parallelism, perpendicularity and angle measure.

Core Standards · Students can and do:

a. Draw points, lines, line segments, rays and angles; identify these in geometric figures.
b. Associate angles of a quarter turn (subtending ¼ of a circle) with angle measure 90°, a half turn (½ of a circle) with angle measure 180°, ¾ turn (¾ of a circle) with angle measure 270°, and a full turn (complete circle) with angle measure 360°.[42]
c. Draw perpendicular and parallel lines; identify these in geometric figures.
d. Identify right angles and angles smaller than/greater than a right angle in geometric figures; recognize right triangles.
e. Given a quadrilateral, say whether it is a square, whether it is a rectangle, and whether it is a parallelogram (with an understanding that a given shape may fit more than one category).

[41] using one-digit or two-digit numbers times two-digit numbers
[42] From Singapore Primary 4

Mathematics: Fifth Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Fractions

Core Standards · Students understand that:

1. Fractions a/b and $(n\times a)/(n\times b)$ are equal: for $1/b$ is n copies of $1/(n\times b)$, so a/b is $n \times a$ copies of $1/(n\times b)$. Example: $1/3$ is 4 copies of $1/12$, so $2/3$ is 8 copies of $1/12$; thus $2/3 = 8/12$.
2. Fractions can be added or subtracted by replacing each with an equal fraction so that the resulting fractions have the same denominator. Example: $2/3 + 5/4 = 8/12 + 15/12 = 23/12$.
3. Multiplying unit fractions gives a new unit fraction with denominator equal to the product of the initial denominators. For example, $1/3 \times 1/2 = 1/(3\times 2)$. The product $1/3 \times 1/2$ is 1 part when a whole of size $1/2$ is divided into 3 parts, i.e. it is "$1/3$ of $1/2$."[43]
4. Multiplying unit fractions can be extended to multiplying fractions in general. For example, $2/3 \times 4/5$ can be seen as 2 groups of 4 unit fractions $1/15$, hence the product is $8/15$.[44] The product $2/3 \times 4/5$ is 2 parts when a whole of size $4/5$ is divided into 3 parts, i.e. it is "$2/3$ of $4/5$."[45]
5. Dividing a unit fraction $1/b$ by a whole number n gives a unit fraction with denominator $n \times b$, because when $1/b$ is divided into n equal parts, the size of each part is $1/(n\times b)$. For example, $1/3 \div 2 = 1/6$.
6. Dividing a whole number n by a unit fraction $1/b$ gives a whole number $n \times b$, because, as there are b units of $1/b$ in 1, there are $n \times b$ units of $1/b$ in n. For example, $2 \div 1/3 = 6$.
7. A mixed number stands for the sum of its whole number part and a fractional part less than 1. A mixed number can be written as a fraction greater than 1, such as $17/5$. This equivalence can be shown using area, length, and number line models.
8. The ratio of two whole number quantities a and b, written $a{:}b$ or a/b, is a multiplicative comparison telling how much of one quantity there is for a given amount of the other, or how many times as much one is than the other.[46]

Core Standards · Students can and do:

a. Use area models and length models (such as strip drawings or the number line) to represent multiplication of fractions, division of unit fractions by whole numbers, and division of whole numbers by unit fractions.
b. Multiply fractions, divide unit fractions by whole numbers, and divide whole numbers by unit fractions, and solve word problems involving these operations.
c. Divide whole numbers by single digit decimals by seeing that they are fractions with denominator 10 or 100.
d. Rename fractions and mixed numbers to equivalent forms and identify equivalent fractions.
e. Compare and order fractions and mixed numbers with like or unlike denominators, including in contextual situations, using the fractions themselves, strip drawings or number line representations, and area models. Describe the size of fractional quantities with reference to the problem situation.
f. Make tables of equal ratios relating whole number quantities, and find missing values in the tables. Plot pairs of values on the coordinate plane. Example

[43] On the number line, $1/n \times 1/d$ is 1 part when the interval from 0 to $1/d$ is divided into n parts. This is the same as 1 part when the interval from 0 to 1 is divided into $n \times d$ parts, and thus $1/n \times 1/d = 1/n\times d$.
[44] Using the Properties of Arithmetic, $2/3 \times 4/5 = (2 \times 1/3) \times (4 \times 1/5) = (2 \times 4) \times (1/3 \times 1/5) = (2 \times 4) \times 1/3\times 5 = 2\times 4/3\times 5$.
[45] On a number line, $m/n \times c/d$ means m parts when the interval from 0 to c/d is divided into n parts.
[46] For example, in a mixture of 5 cups of flour and 2 cups of sugar, the ratio is 5 cups flour to 2 cups sugar. There is $5/2$ times as much flour as sugar (equivalently, 2 $1/2$ times as much or 2.5 times as much).

Cups of Flour	5	10	?	20
Cups of Sugar	2	?	6	8
Flour:Sugar (fraction form)	$5/2$	$10/4$	?	$20/8$
Flour:Sugar (decimal form)	?	?	?	?

Hours of Snowfall	5	10	?	20
Inches of Snow	2	?	6	8
Inches:Hours (fraction form)	$2/5$	$4/10$	?	$8/20$
Inches:Hours (decimal form)	?	?	?	?

Base Ten Computation

Core Standards · Students understand that:

1. The standard division algorithm is based on successively finding the largest single digit multiple of the divisor that is less than the dividend, regrouping to the next lower unit if necessary, and then subtracting the multiple and repeating to find the next digit in the quotient.
2. The division algorithm can be used to express a fraction in decimal form by carrying the division into the decimal places.
3. The features of the place value system for whole numbers extend to the decimal positions and the combined system is symmetric around the ones place.
4. In adding or subtracting decimal numbers, one operates separately with the units of each size, except when regrouping is needed; the scheme for regrouping is the same at each place, because each unit is composed of ten of the next smaller unit.
5. Numbers in decimal notation can be shown on the number line by dividing and sub-dividing the unit intervals as many times as needed to locate the number. This process can be visualized as zooming in on the number line.

Core Standards · Students can and do:

a. Divide two, three and four digit numbers by two digit numbers, with remainder, using an algorithm based on place value and regrouping, such as the standard algorithm. In the case of remainders, express results in the form of an equation, as in $145 = 11 \times 13 + 2$.
b. Understand very large and very small numbers (from millionths to hundreds of millions); round very large numbers.
c. Quickly find 0.1 more than a number and less than a number; 0.01 more than a number and less than a number; and 0.001 more than a number and less than a number.
d. Add and subtract decimals using an algorithm based on place value and regrouping, such as the standard algorithm, and solve problems involving these operations.
e. Write fractions in decimal notation for denominators 2, 3, 4, 5, 6, 8, 10 and 100.
f. Explain why strategies and algorithms for decimals work, using place value and the Properties of Arithmetic (including explanations supported by drawings or objects).

Quantity and Measurement

Core Standards · Students understand that:

1. The volume of a solid figure is a measure of how much space it encloses. A cube with side length 1 unit is said to contain "one cubic unit" of volume. The volume of a solid figure can be measured (expressed numerically) by the number of cubic units that fit inside it with no gaps or overlaps.
2. Packing a rectangular prism with unit cubes and decomposing the prism into layers shows that a rectangular prism ℓ units long by w units wide by h units tall contains a volume $V = \ell \times w \times h$ cubic units. The base of the

prism has area $A = \ell \times w$ square units, and the prism can be viewed as h layers, each containing $\ell \times w$ cubic units, so the volume of the prism can also be expressed as $V = A \times h$ cubic units.
3. Volume is additive: If a solid figure is decomposed into several pieces, then the volume of the whole figure can be found by adding the volumes of the pieces (expressed in common units).
4. Quantities with like units can be added or subtracted giving a sum or difference with the same unit; quantities with unlike units can be multiplied or divided giving products or quotients with derived units.
5. The ratio of a length, area or amount to another length, area or amount is the same regardless of the size of the unit used for measurement.
6. The number line is a scale that can be used to show units such as pounds, liters, etc.

Core Standards · Students can and do:

a. Measure and compute whole-cubic-unit volumes for rectangular prisms and for objects well described as rectangular prisms.
b. Convert among different-sized standard measurement units within a given measurement system (e.g. feet to yards, centimeters to meters) and use conversion to solve story problems.
c. Form ratios of lengths, areas, and other quantities, including when quantities being compared are measured in different units.
d. Solve word problems involving addition, subtraction, multiplication and/or division using quantities expressed as whole numbers, fractions, or decimals with measurement units.
e. Solve multi-step problems involving units of weight, capacity, money, volume and area.

Coordinate Geometry

Core Standards · Students understand that:

1. A pair of perpendicular number lines ("axes") defines a coordinate system. A given point in the plane has a separate position along each of the two axes; the two positions of the point are called its coordinates.
2. Graphs on coordinate axes can be used to make sense of relationships among quantities in complex problems.

Core Standards · Students can and do:

a. Graph points in the first quadrant the coordinate plane, and read off the coordinates of graphed points.[47]
b. Determine the lengths of horizontal and vertical segments in the plane, given the coordinates of their endpoints.
c. ♦ Collect data on continuous covarying quantities and display the data in a line graph with broken lines; distinguish bar graphs from line graphs; ask and answer questions from line graphs, including comparisons of ratios.

Statistics

Core Standards · Students understand that:

[47] The axes should sometimes represent dimensioned quantities, and the units of measure should not always be the same for both axes. Coordinates may be whole numbers, fractions or decimals.

1. Data are collected purposefully to answer a predefined question (e.g., "How tall are the fifth graders in our school?")
2. A set of data typically shows variability—not all of the values are the same—and yet the values also typically show some tendency to cluster. Identifying a "center" for a data set is a way to describe its many values using a single number.
3. The median is a measure of center in the sense that approximately half the data values are less than median, while approximately half are greater.
4. Variation in a data set can be measured by the range and by typical deviations from the center.

Core Standards · Students can and do:

a. Collect data to answer a predefined question about a measurement quantity. Make a dot plot to display the data, and describe the data using the median and typical deviations from the it.

Mathematics: Sixth Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Ratios and Proportional Relationships

Core Standards · Students understand that:

1. Multiplicative comparisons can be extended from whole numbers to fractions and decimals. When the ratio q/m is formed, or when q is r times as much as m, the numbers q, r and m can be fractions or decimals.
2. p% of a quantity means $p/100$ times as much as the quantity. The number p can be a fraction or decimal, as in 3.75%.
3. A unit rate is the multiplicative factor relating the two quantities in a ratio. Two quantities q and m can be compared by $q = r \times m$, where the unit rate r tells how much q per m.
4. Given two quantities in a ratio (e.g. distance and time), finding the unit rate produces a new type of quantity (e.g. speed).

Core Standards · Students can and do:

a. Solve for an unknown quantity in a problem involving two equal ratios.
b. Find a percentage of a quantity; solve problems involving finding the whole given a part and the percentage.
c. Solve unit rate problems including unit pricing and constant speed. (See table.)

$D = s \times T$	$D \div T = s$	$D \div s = T$
A car driving at a speed of 30 miles per hour for 6 hours travels a distance of 180 miles.	If a car drives 180 miles for 6 hours at a constant speed, that speed is 30 miles per hour.	When a car drives 180 miles at a speed of 30 miles per hour, the trip takes 6 hours.

d. Represent unit rate problems on a coordinate plane where each axis represents one of the two quantities involved, and find unit rates from a graph. Explain what a point (x, y) means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

The Number System

Core Standards · Students understand that:

1. The Properties of Arithmetic govern operations on all numbers.
2. Division of fractions follows the "invert and multiply" rule because multiplication and division are inverse operations. For example, $(2/3) \div (5/7) = 14/15$ because $(14/15) \times (5/7) = 2/3$.
3. Every nonzero fraction has a unique multiplicative inverse,[48] namely its reciprocal. Division can be defined as "multiplying by the multiplicative inverse." Then $(2/3) \div (5/7) = 14/15$ because the division symbol indicates multiplication by the multiplicative inverse.
4. A two-sided number line can be created by reflecting the fractions across zero. Numbers located to the left of zero on the number line are called negative numbers and are labeled with a negative sign.

[48] Glossary: Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $3/4$ and $4/3$ are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

5. Two different numbers, such as 7 and –7, that are equidistant from zero are said to be opposites of one another. The opposite of 7 is –7 and the opposite of –7 is 7. The opposite of the opposite of a number is the number itself. The opposite of 0 is 0. The operation of attaching a negative sign to a number can be interpreted as reflecting the number across zero on the number line.
6. The absolute value of a number is its distance from zero on the number line. For any positive number q, there are two numbers whose absolute value is q, namely q and $-q$.
7. The absolute value of a signed quantity (e.g. account balance, elevation) tells the size of the quantity irrespective of its sense (debit or credit; above or below sea level).
8. Comparison of numbers can be extended to the full number system. The statement $p > q$ means that p is located to the right of q on the number line, while $p < q$ means that p is located to the left of q on the number line. The statement $p > q$ does not mean $|p| > |q|$.

Core Standards · Students can and do:

a. Divide fractions, and divide finite decimals by expressing them as fractions.
b. Solve problems requiring arithmetic with fractions presented in various forms, converting between forms as appropriate and estimating to check reasonableness of answers.
c. Find and position rational numbers[49] on the number line.
d. Use rational numbers to describe quantities such as elevation, temperature,[50] account balance and so on. Compare these quantities using > and < symbols and also in terms of absolute value.
e. Graph points and identify coordinates of points on the Cartesian coordinate plane in all four quadrants.

Statistics

Core Standards · Students understand that:

1. The mean is a measure of center in the sense that it is the balance point; the mean is the value each data point would take on if the total value of all the data points were redistributed fairly.
2. When the mean and median of a data set differ substantially, both measures should be provided, and the difference explained in terms of the data values.

Core Standards · Students can and do:

a. Collect data to answer a predefined question about a measurement quantity. Make a dot plot to display the data, and describe the data using measures of center and measures of variation.[51]

Geometry

Core Standards · Students understand that:

1. Triangles and parallelograms can be dissected and reassembled into rectangles with the same area; this leads to a formula for area in terms of base and height.
2. Polygons can be dissected into triangles in order to find their area.

Core Standards · Students can and do:

[49] Glossary: Rational number. A number expressible in the form a/b for integers a and $b \neq 0$. The rational numbers include positive and negative integers, positive and negative fractions, and 0.
[50] A caution for temperature problems: The rational numbers are not a good model for a temperature scale. There is no temperature that solves the equation $T + 1000^{\circ}C = 0$.
[51] Data sets should include fractional values at this grade but not negative values.

a. Find the area of right triangles, other triangles, special quadrilaterals, and polygons (by dissection into triangles and other shapes).
b. Find surface area of cubes, prisms and pyramids (include the use of nets to represent these figures).
c. Solve problems involving area, volume and surface area of objects.
d. Examine the relationship between volume and surface area. Exhibit rectangular prisms with the same surface area and different volume, and with the same volume and different surface area.
e. Use exponents and symbols for square roots and cube roots to express the area of a square and volume of a cube in terms of the side length, and to express the side length in terms of the area or volume.

Expressions and Equations

Core Standards · Students understand that:

1. A number that is the result of a sequence of operations with other numbers can be expressed in different ways using conventions about order of operations and parentheses, rules for working with fractions, and the Properties of Arithmetic. All such expressions are equivalent.
2. A letter is used to stand for a number in an expression in cases where one doesn't know what the number is, or where, for the purpose at hand, it can be any number in the domain of interest. Such a letter is called a variable.
3. An equation is a statement that two expressions are equal, and a solution to an equation is a value of the variable (or a set of values for each variable if there is more than one variable) that makes the equation true.

Core Standards · Students can and do:

a. Represent an unknown number using a letter in simple expressions such as $y + 2$, $y - 3$, $6 + y$, $5 - y$, $3y$, $y/2$, and $(3 \pm y)/5$.
b. Interpret $3y$ as $y + y + y$ or $3 \times y$, $y/2$ as $y \div 2$ or $1/2 \times y$, $(3 \pm y)/5$ as $(3 \pm y) \div 5$ or $1/5 \times (3 \pm y)$.[52]
c. Evaluate simple expressions when values for the variables in them are specified (exclude expressions with a variable in denominator).
d. Choose variables to represent quantities in a word problem and construct simple equations to solve the problem by reasoning about the quantities.
e. Solve equations of the form $x + p = q$ (for $p < q$) and $px = q$ where p and q are fractions.

[52] From Singapore Secondary 1

Mathematics: Seventh Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Ratios and Proportional Relationships

Core Standards · Students understand that:

1. Two variable quantities x and y are said to be proportional to one another if the ratio x/y is always equal to the same quantity k, so that $y = kx$. The constant k is the unit rate, and tells how much of y per unit of x.

Core Standards · Students can and do:

a. Compare proportional relationships represented in different ways (e.g. compare a graph to an equation to determine which of two objects has greater speed).
b. Decide whether two quantities that vary together have a proportional relationship, analyze proportional relationships using the unit rates that characterize them, and solve word problems involving proportional relationships.
c. Plot pairs (x, y) from a proportional relationship $y = kx$, and pass a straight line through them and the origin. Observe that increases in y are proportional to increases in x, and calculate [increase in y]/[increase in x] $= k$.

The Number System

Core Standards · Students understand that:

1. On the number line, the sum $p + q$ is defined to be the number lying distance $|q|$ from p, to the right of p if q is positive and to the left of p if q is negative. A number and its opposite are additive inverses (add to zero).[53]
2. Sums of signed numbers can be computed using the Properties of Arithmetic.[54]
3. The additive inverse of a sum is the sum of the additive inverses.[55]
4. Subtraction is defined as adding the additive inverse. This definition of subtraction allows subtraction of rational numbers and agrees with previous understandings of subtraction with positive numbers.[56] On the number line, the difference $p - q$ lies distance $|q|$ from p, to the left of p if q is positive and to the right of p if q is negative.
5. The absolute value of $p - q$ equals the distance between p and q on the number line.
6. Products of signed numbers can be computed using the Properties of Arithmetic.[57] In particular, multiplying a number by -1 produces its additive inverse.[58]

[53] Glossary: Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $^3/4$ and $-^3/4$ are additive inverses of one another because $^3/4 + (-^3/4) = (-^3/4) + ^3/4 = 0$.
[54] For example, 7 + (−3) = 4 because 7 + (-3) = (4 + 3) + (−3) = 4 + [3 + (−3)] = 4 + [0] = 4. And (−2) + (−3) = −5 because 5 + [(-2) + (-3)] = (2 + 3) + [(-2) + (-3)] = [2 + (-2)] + [3 + (-3)] = [0] + [0] = 0 so (-2) + (-3) is the additive inverse of 5, that is −5.
[55] For example, −(6 + −2) = (−6) + 2 because [6 + (−2)] + [(−6) + 2] = [6 + (−6)] + [(−2) + 2] = [0] + [0] = 0.
[56] For example, the subtraction 7 – 3 means 7 plus the additive inverse of 3, i.e. 7 + (–3), which equals 4. The subtraction 9 – (–4) means 9 plus the additive inverse of –4, i.e. 9 + 4, which equals 13.
[57] For example, (−1) × (−1) = 1 because (−1) + (−1) × (−1) = 1 × (−1) + (−1) × (−1) = [1 + (−1)] × (−1) = 0 × (−1) = 0.
[58] Because $(-1) \times a + a = (-1) \times a + (1) \times a = [(-1) + (1)] \times a = 0 \times a = 0$.

7. Every nonzero rational number has a multiplicative inverse. Division of rational numbers is defined as multiplying by the multiplicative inverse.
8. The operation of adding the rational number q to points on the number line is a translation; it shifts points to the right if $q > 0$, to the left if $q < 0$, and not at all if $q = 0$. The operation of adding $-q$ undoes the operation of adding q.
9. The operation of multiplying points on the number line by a positive rational number k is a dilation; it scales points further away from zero if $k > 1$, closer to zero if $k < 1$, and not at all if $k = 1$. The operation of multiplying by $1/k$ undoes the operation of multiplying by k.

Core Standards · Students can and do:

a. Explore and explain with number lines the rules for adding rational numbers, e.g., $r + s = s + r$; $r + (-s) = r - s$; $p - (q + r) = p - q - r$.
b. Use the rules of arithmetic to explore and explain with specific numbers the rules for multiplying rational numbers, e.g., 4×-5 is -5 added to itself 4 times, so equal to -20; $-3 \times (-2 + 2) = -3 \times 0 = 0$, so $-3 \times -2 = -(-3)\times 2 = -(-6) = 6$.
c. Add and subtract rational numbers, and use these operations to solve word problems (including signed quantities such as elevation, temperature, account balance, and so on).
d. Multiply and divide rational numbers, and use these operations to solve word problems (including signed quantities).

Expressions and Equations

Core Standards · Students understand that:

1. Expressing a quantity in different forms serves a purpose in analyzing quantitative situations.
2. The distributive property can be used in two directions, both to expand linear expressions, and to factor a sum of terms with a common factor.

Core Standards · Students can and do:

a. Construct algebraic expressions for simple real-world situations and generate equivalent expressions to interpret their meaning (e.g., $P + 0.05P = 1.05P$ means that "increase by 5%" is the same as "multiply by 1.05").
b. Generate equivalent expressions from a given expression, including putting linear expressions in standard form and taking out a common factor. Include expressions involving negative numbers and exponents 2 and 3.
c. Solve multi-step word problems that lead to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers, by undoing the operations involved in producing the expression on the left, using additive and multiplicative inverses.
d. Solve simple absolute value equations of the form $|x + h| = j$ and $|x - h| = j$, where h and j are integers.
e. Read the structure in a numerical expression at a level necessary to enter it into a calculator or spreadsheet, making use of parentheses and the conventions on order of operations.

Statistics

Core Standards · Students understand that:

1. In addition to measurement variability, another source of variation in data is randomness.

Core Standards · Students can and do:

a. Collect experimental or simulation data from repeated random trials. Make a histogram showing absolute frequencies and a bar graph of relative frequencies. Discuss the patterns and make predictions for further experiments or simulations.

Probability

Core Standards · Students understand that:

1. Chance events fall along a spectrum: nearly impossible | unlikely | neither likely nor unlikely | likely | nearly certain.
2. Probability is a quantitative measure of likelihood. Probabilities are numbers lying between 0 and 1, with 0 representing impossible and 1 representing certain (in the case of a finite sample space).
3. The experimental probability of a specified outcome is the observed fraction of the outcome in a data set collected from a process involving randomness or chance.
4. In a random process, the individual outcomes are unpredictable, but patterns may emerge after repeated trials. Experimental probabilities in random experiments tend to approach stable values as more and more data is generated.
5. In a theoretical probability model, the set of distinct possible outcomes for a random experiment is called the sample space. An event is a set of sample points; a sample point may belong to several events. A specified event occurs in some fraction of the sample space. This fraction is called the theoretical probability of the event.
6. When computing theoretical probabilities, all members of the sample space are assumed equally probable. Theoretical probabilities will not match long-run experimental probabilities if this assumption is inappropriate (e.g., as in the case of a loaded die).

Core Standards · Students can and do:

a. Compute experimental probabilities from data sets, including data sets generated by simulations or sampling experiments.
b. Compute experimental probabilities to estimate theoretical probabilities when no theoretical probability model is apparent.
c. Represent sample spaces for one-stage random experiments; identify members of the sample space in which specified events occur.
d. Use a theoretical probability model to compute theoretical probabilities for one-stage random experiments, expressing theoretical probabilities as fractions, decimals and percents.
e. Compare experimental probabilities to theoretical probabilities for one-stage random experiments, examining and if feasible revising the assumptions of the theoretical model when the two conflict.

Geometry

Core Standards · Students understand that:

1. Two polygons are congruent[59] if and only if there is a correspondence between vertices so that the corresponding sides are equal and the corresponding angles are equal.
2. A plane or solid figure is similar to another if the second can be obtained from the first by a similarity transformation.[60] All ratios of lengths in the second figure to corresponding lengths in the first figure are equal to the scale factor of the dilation.

[59] Glossary: Congruent. Two plane or solid figures are *congruent* if one can be obtained from the other by a sequence of rigid motions (rotations, reflections, and translations).
[60] Glossary: Similarity transformation. A rigid motion followed by a dilation. Glossary: Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

3. Congruent figures have the same area or volume. A similarity transformation with a scale factor of k leaves angle measures unchanged, changes lengths by a factor of k, changes areas by a factor of k^2, and changes volumes by a factor of k^3.
4. Given a line in the coordinate plane not parallel to either axis, any two right triangles with legs parallel to the axes and hypotenuse on the given line are similar, and so the slope of the line (rise over run) is the same regardless of which two distinct points are used to compute it.

Core Standards· Students can & do:

a. Solve problems involving similar triangles and scale drawings (including computing actual lengths, areas and volumes from a scale drawing and reproducing a scale drawing at a different scale).
b. Explore using hands-on activities the area of non-rectangular figures and the perimeter of curvilinear figures, and the fact that a dilation of the plane changes areas by the square of the scale factor.[61]
c. Use scale factors to find lengths and areas of similar figures, including an informal derivation of the formulas relating the area, radius and circumference of a circle.
d. Give an explanation of why the volume of a cylinder is the area of the base times the height, using informal arguments involving slices.
e. Use coordinate grids to transform figures and to predict the effect of dilations, translations, rotations and reflections.
f. Use two-dimensional representations of three-dimensional objects (schematics, assembly instructions, perspective drawings and multiple views) to solve problems.
g. Explore three-dimensional figures formed by translations and rotations of plane figures through space.
h. Sketch and describe cross-sections of cones, cylinders, pyramids and prisms.

[61] Include using grids of squares with fractional side lengths to estimate area, and measuring the length of strings wrapped around the perimter.

Mathematics: Eighth Grade

Developing Coherent Understanding

[Temporarily removed for editing.]

Functions and the Situations They Model

Core Standards · Students understand that:

1. A function is a rule, often defined by an expression, that states a relationship between the values of two variable quantities.
2. A linear function models a situation where the change in one quantity is proportional to the corresponding change in the other quantity. The constant of proportionality, m, is the rate of change of the function. If x is the input and y is the output then the function is defined by $y = mx + b$ for some constant b, which is called the initial value of the function (the value of the function when x is 0).
3. The graph of a linear function $y = mx + b$ is a straight line, and the slope of the line is the function's rate of change.
4. The problem of finding where two linear functions have the same output value for a common input value leads to an equation in one variable; the solution or solutions (if any) can be visualized as the input value(s) where the graphs of the functions intersect.
5. A linear equation in one variable can be solved by successively transforming it into simpler equations with the same solutions using the Properties of Arithmetic and the Properties of Equality, until an equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Core Standards · Students can and do:

a. Compare features of two or more functions that may be presented in different representations (as formulas, graphs, tables of values, or verbally).
b. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship; from two (x, y) values (including reading these from a table); or from a graph.
c. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
d. Solve linear equations with rational number coefficients, including equations that require expanding expressions using the distributive law and collecting like terms.

The Number System

Core Standards · Students understand that:

1. The number line has numbers that are not rational, such as 2π or $2 + \sqrt{3}$, called irrational numbers.
2. An irrational number can be approximated to arbitrary precision by rational numbers.
3. If $n > 0$ is an integer and $\sqrt{n}$ is not an integer, then $\sqrt{n}$ is irrational. If q is rational and r is irrational, then $q + r$ is irrational, and so is qr provided $q \neq 0$.

Core Standards · Students can and do:

a. Use rational approximations to compare the size of irrational numbers, locate them approximately on a number line and estimate the value of expressions (e.g. π^2).

Geometry

Core Standards · Students understand that:

1. Angle measures formed by a configuration of lines in a plane can often be deduced from other angle measures (e.g., vertically opposite angles, angles produced when a transversal line cuts two parallel lines).
2. The side lengths of a right triangle are related by the Pythagorean theorem.

Core Standards · Students can and do:

a. Explore and explain by hands-on activities facts about the angle sum of triangles, exterior angles, and alternate interior angles of parallel lines. Use these facts to determine the angle sum of interior angles of convex polygons, and the angle sum of exterior angles of convex polygons.[62]
b. Explore and explain using hands-on activities: parallel lines in space, line perpendicular to a line through a given point, lines perpendicular to a plane, lines parallel to a plane, the plane passing through three given points, and the plane perpendicular to a given line at a given point.
c. Use facts about angles to write and solve simple equations for an unknown angle in a figure.
d. Explain a proof of the Pythagorean theorem.[63]
e. Use the Pythagorean theorem to determine missing side lengths in right triangles and to solve problems in two and three dimensions.
f. Use the Pythagorean theorem to find the distance between two points in a coordinate system.
g. Draw (freehand, with ruler and protractor, and with technology) geometric shapes from given conditions. (Focus on constructing triangles from three measures of angles or sides, noticing when the triangle is uniquely defined, ambiguous, or impossible.)
h. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc): copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
i. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Statistics

Core Standards · Students understand that:

1. Scatterplots for bivariate continuous data may reveal patterns of association between two quantities. This kind of relationship between quantities is not a functional relationship—and yet, a function might be a valuable way to describe a statistical relationship.

Core Standards · Students can and do:

a. Construct and interpret scatterplots for bivariate measurement data.
b. Describe patterns that appear in scatterplots, such as clustering, outliers, positive/negative association, linear association, nonlinear association.
c. For scatterplots that suggest a linear association, model the relationship with a linear function using an informal fitting procedure. Use the model function to solve problems in the context of the data,

[62] Use physical models, transparencies, or dynamic geometry software to make rigid motions and give informal arguments, for example, arrange three copies of the same triangle so that the three angles form a line, and give an argument in terms of transversals why this is so.
[63] For example, by the method of right triangles in a square.

interpreting the slope/rate of change and intercept/initial value. Informally assess the goodness of the model by judging the closeness of the data points to the graph of the function.

Probability

Core Standards · Students understand that:

1. The framework for theoretical probability models is the same for compound events as for simple events: the theoretical probability is the fraction of the sample space in which the compound event occurs.

Core Standards · Students can and do:

a. Compute experimental probabilities from data sets, including data sets generated by simulations or repeated sampling experiments.
b. Compute experimental probabilities to estimate theoretical probabilities of compound events when no theoretical probability model is apparent.
c. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams; identify members of the sample space in which specified events occur.
d. Compute theoretical probabilities for compound events by counting members of the sample space.
e. Compare experimental probabilities to theoretical probabilities for multi-stage random experiments, examining the assumptions of the theoretical model when the two conflict.

Mathematics: High School—Expressions

A Coherent Understanding of Expressions

[Final draft of CCR narrative goes here.]

Seeing structure in expressions

Core Standards · Students understand that:

1. Different forms of expression for functions reveal different properties of the function; a purpose in transforming expressions is to find those properties.

   For example, factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals the maximum or minimum of the function; the expression 1.15^t can be rewritten in the form $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate monthly interest rate if the annual rate is 15%.

2. The laws of exponents for whole number exponents follow from an understanding of exponents as indicating repeated multiplication, and from the associative property of multiplication.

3. The interpretation of zero, fractional and negative exponents follows from extending the laws of exponents to those values.

   For example, since $(x^{1/3})^3 = x^{(1/3)\cdot 3} = x^1 = x$, $x^{1/3}$ is the cube root of x.

4. Complex expressions can be interpreted by "chunking": temporarily viewing a part of the expression as a single entity.

Core Standards · Students can and do:

a. Factor, expand, and complete the square in quadratic expressions.
b. Use chunking to see expressions in different ways that suggest ways of rewriting them.

   For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

c. Rewrite expressions using the laws of exponents.

   For example $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.

d. Use the laws of exponents to interpret expressions for exponential functions, recognizing fractional exponents as indicating roots of the base and negative exponents as indicating the reciprocal of a power.

   For example, identify the relative rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.2)^{10t}$, $y = (1.01)^{t/12}$, and recognize that any non-zero number raised to the 0 power is 1 (for example, $12(1.05)^0 = 12$). Avoid common errors such as confusing as $6(1.05)^t$ with $(6\cdot 1.05)^t$ and $5(0.03)^t$ with $5(1.03)^t$.

e. Given an expression for an exponential function, identify whether it represents exponential growth or decay.
f. Using a method such as the factorization $(x^n - 1) = (x-1)(x^{n-1} + \ldots + 1)$ where n is a whole number, prove the formula for the sum of a geometric series, and use the formula to solve problems.

   Include problems involving compound interest and mortgage payments.

The arithmetic of polynomials and rational functions

Core Standards · Students understand that:

1. Polynomials form a system analogous to the integers, closed under the operations of addition, subtraction, and multiplication.

2. A polynomial of degree n has n complex roots, where roots are counted according to multiplicity.
3. For a polynomial $p(x)$, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
4. The Binomial Theorem gives the expansion of $(x + a)^n$ in powers of x for a whole number n and a real number a, with coefficients determined for example by Pascal's triangle. The Binomial Theorem can be proved by mathematical induction.
5. Rational functions are fractions whose numerator and denominator are polynomials, and the rational functions are closed under the operation of division by a nonzero rational function.

Core Standards · Students can and do:

a. Add, subtract and multiply polynomials.
b. Identify zeros of polynomials when suitable factorizations are available, and graph polynomials.
c. Transform simple rational functions using the Properties of Arithmetic and the rules for operations on fractions.
d. Identify zeros and asymptotes of rational functions, when suitable factorizations are available, and graph rational functions.
e. Divide polynomials by monomials

Mathematics: High School—Equations

A Coherent Understanding of Equations

[Final draft of CCR narrative goes here.]

Building equations to model relations between quantities

Core Standards · Students understand that:

1. Choosing a unit for a general quantity (e.g. length) establishes a correspondence between specific instances of the quantity (e.g. lengths of specific objects) and numbers called coordinates.
2. A relation between two quantities can be represented by an equation in variables representing coordinates for the quantities; by a graph on a pair of axes marked with units for the quantities; and by a table of coordinate pairs from the relation. The graph and the table show pairs that are solutions to the equation.

Core Standards · Students can and do:

a. Build equations to express relations between quantities and solve problems.

   Include equations arising from situations involving linear, quadratic, simple rational, and exponential functions.

b. Rearrange formulas to isolate a quantity of interest.
c. Build systems of equations and solve problems involving systems of equations.

Reasoning with equations and inequalities

Core Standards · Students understand that:

1. To solve an equation algebraically, one assumes it is true and deduces the solutions, often in steps that replace it with a simpler equation whose solutions include the solutions of the original one.
2. Adding a number to both sides of an equation, or multiplying both sides by a nonzero number, leads to an equation that has exactly the same solutions as the original.
3. If the product of two numbers is zero, then at least one equals zero, and conversely. This principle is the basis for solving equations by factoring.
4. Multiplying both sides of an equation by an expression that can be zero for certain values of the variables in it, or squaring both sides of an equation, can lead to an equation that has more solutions than the original. Evaluating these in the original equation eliminates extraneous solutions.
5. The method of completing the square can transform any quadratic equation in x into an equivalent equation of the form $(x - p)^2 = q$. This leads to the quadratic formula.
6. Equations not solvable in one number system may have solutions in a larger number system.
7. Equations of the form $f(x) = g(x)$ can be solved graphically by finding the intersections (if any) of the graphs of $f(x)$ and $g(x)$.
8. The relationship between a function f and its inverse (if it has one) can be used to solve equations of the form $f(x) = c$. For example, a logarithmic function can help solve exponential equations, and an inverse trigonometric function can help solve trigonometric equations.
9. Given a system of linear equations, adding a multiple of one equation to another produces a system with the same solutions. This principle, combined with principles already encountered with equations in one variable, allows for the simplification of systems.

10. The solutions to an equation in two variables form a graph—a set of points, often a curve or a line, in the coordinate plane.
11. The solutions to two equations in two variables (if any) can be visualized as the points of intersection of their graphs, because those points satisfy both equations simultaneously.
12. The solution to a system of inequalities in two variables can be visualized as the intersection of the regions in the plane defined by the inequalities.

Core Standards · Students can and do:

a. Solve simple rational and radical equations, noting and explaining extraneous solutions.
b. Solve quadratic equations over the real numbers by completing the square, using the quadratic formula and factoring.
c. Solve linear inequalities in one variable and graph the solution set on a number line.

Emphasize solving the associated equality and determining on which side of the solution of the associated equation the solutions to the inequality lie.

d. Solve linear systems of equations algebraically, focusing on pairs of linear equations in two variables.
e. Graph a system of two linear or quadratic equations in two unknowns and estimate the solution from a graph.
f. Graph the solution set of a linear inequality in two variables.
g. Use the properties of logarithms to solve equations involving exponential functions.
h. Use inverse trigonometric functions to solve equations of the form $A\sin(Bx + C) = D$.
i. Find complex roots of quadratic equations.
j. Solve a system of two quadratic equations in two unknowns.

Mathematics: High School—Functions

A Coherent Understanding of Functions

[Final draft of CCR narrative goes here.]

Interpreting functions

Core Standards · Students understand that:

1. The domain of a function is the set of its inputs, and the range is the set of its outputs.
2. Function notation uses a letter to stand for a function. If f is a function and x is a number in its domain, then $f(x)$ indicates the output of f corresponding to the input x.
3. Functions can be described by key characteristics, including: zeros; vertical intercept; extreme points; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
4. Linear, quadratic and exponential functions are defined by expressions that have forms specific to each type, in which the parameters can often be interpreted in terms of characteristics of the graph.
5. An equation in two variables implicitly expresses one variable as a function of the other if there are no points on the graph having the same value of the first variable but different values of the second.
6. When x is a power of ten, the common logarithm $\log(x)$ tells the exponent. When x lies between 10^n and 10^{n+1}, $\log(x)$ lies between n and $n+1$.

Core Standards · Students can and do:

a. Describe qualitatively the functional relationship between two quantities by reading a graph; e.g., where the function is increasing or decreasing, what its long run behavior appears to be, and whether it appears to be periodic.
b. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
c. Compare values and properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions).
d. Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
e. Describe the qualitative behavior of common types of functions using graphs and tables.

Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Use technology to explore the effects of parameter changes on the graphs of linear, power, quadratic, square root and cube root, polynomial, simple rational, exponential, logarithmic, sine and cosine, absolute value and step functions.

f. Interpret the parameters in the general expressions for linear, quadratic, and exponential functions, and draw conclusions about the parameters by inspection of the graph.
g. Given a function f, and given a constant c, evaluate $f(c)$ if possible and find solutions to $f(x) = c$ (if they exist). Where appropriate, relate the possibility of evaluation to the domain and the existence or nonexistence of solutions to the range.

Building functions

Core Standards · Students understand that:

1. Varying a parameter in the general expression for a linear, quadratic or exponential function can (often) be interpreted as performing a geometric transformation on the graph. This can be used to adjust a function to model a particular situation.

2. Composing a function f with a function g creates a new function called the composite function—for an input number x, the output of the composite function is $f(g(x))$.
3. The inverse of a function "undoes" what the function does; that is, composing the function with its inverse in either order returns the original input.
4. Sequences are functions whose domain is the whole numbers, and they can be defined recursively as well as explicitly. Arithmetic sequences are linear functions and geometric sequences are exponential functions.

Core Standards · Students can and do:

a. Make graphs of linear, quadratic, cubic, absolute value and exponential functions, and, given the graph of one of these types, identify the type.
b. Sketch graphs of quadratic functions presented in the form $y = ax^2+bx+c$, $y = a(x-h)^2+k$ and $y = a(x-p)(x-q)$ (without plotting points).
c. Solve problems involving quadratic functions, such as analyzing projectile motion and maximizing profit.
d. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$. Include both positive and negative k; find the value of k given the graphs.
e. Write an expression of the form $a(1+r)^t$ or ab^t for an exponential function to express a constant percent growth rate or a constant growth factor.
f. Evaluate composite functions and compose functions symbolically in simple cases (e.g. one or both functions linear).
g. Read values of an inverse function from a graph or a table, given that the function has an inverse.
h. For linear or simple exponential functions, find a formula for an inverse function by solving an equation.
i. For linear functions or simple exponential functions, verify symbolically by composition that one function is the inverse of another.
j. Write arithmetic and geometric sequences both recursively and in closed form, and translate between the two forms.

Linear vs. exponential behavior

Core Standards · Students understand that:

1. Linear functions grow by equal differences in equal time periods; exponential functions grow by equal factors in equal time periods.
2. The rate of change of a linear function is constant; the rate of change of an exponential function is proportional to the value of the function.
3. Exponential growth eventually outstrips polynomial growth (including, in particular, linear growth).

Students can and do:

a. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
b. Interpret absolute and relative rates of change and use them to make predictions.
c. Identify the initial value and growth or decay rate from a table or graph of an exponential function.
d. Calculate and interpret the growth factor for an exponential function (presented symbolically or as a table) given a fixed time interval. Estimate the growth factor from a graph.
e. Recognize a quantitative relationship as linear or exponential from description of a situation.

Trigonometric functions

Core Standards · Students understand that:

1. The unit circle in the coordinate plane enables one to extend the domains of the sine, cosine and tangent functions of right-triangle trigonometry to the real numbers.
2. Trigonometric functions are periodic by definition, and sums and products of these functions are periodic.
3. Restricting trigonometric functions to a domain on which they are always increasing or always decreasing allows for the construction of an inverse function.

Core Standards · Students can and do:

a. Use radian measure and revisit graphs of trigonometric functions in terms of radians.
b. Use the unit circle to determine geometrically the values of sine, cosine, tangent for multiples of $\pi/4$ and $\pi/3$; commit sines and cosines of principal angles to memory.
c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
d. Solve simple trigonometric equations formally using inverse trigonometric functions;[64] evaluate solutions using technology.
e. Explain relationships between the identity $\sin^2 x + \cos^2 x = 1$, the equation of a circle, and the Pythagorean theorem.
f. Explain proofs of the sine and cosine addition and subtraction formulas.
g. Use trigonometric identities to simplify expressions.
h. Use trigonometric functions to solve problems in science, economics or other fields where periodic phenomena occur.

[64] Solving trigonometric equations by means of the quadratic formula is optional.

Mathematics: High School—Coordinates

A Coherent Understanding of Coordinates.

[Final draft of CCR narrative goes here.]

Expressing geometric properties with equations

Core Standards · Students understand that:

1. The graph of a linear equation is the straight line through any two of its solutions. Conversely, any line is the set of solutions to some linear equation.
2. Two lines with well-defined slopes are parallel if their slopes are equal and perpendicular if their slopes multiply to –1.
3. The equation of a circle can be found using its definition and the Pythagorean theorem.
4. Transforming the graph of an equation by reflecting in the axes, translating parallel to the axes, or applying a dilation to one of the axes correspond to substitutions in the equation.

   For example, reflection in the y axis corresponds to $(x,y) \to (-x,y)$, translation vertically down by three units corresponds to $(x,y) \to (x,y+3)$, and dilating by a factor of 2 parallel to the x-axis corresponds to $(x, y) \to (x/2, y)$.
5. An ellipse is obtained by stretching a circle, leading to an equation of the form $x^2/a^2 + y^2/b^2 = 1$.
6. The formula $A = \pi ab$ for the area of an ellipse can be derived from the formula for the area of a circle.

Core Standards · Students can and do:

a. Write the equation of a line in point-slope form, slope-intercept form, or standard form.
b. Identify parallel and perpendicular lines in a coordinate plane, and use the relationship between slopes of parallel and perpendicular lines to solve problems. Know the equations of vertical and horizontal lines.
c. Find the point on the segment between two given points that divides the segment in a given ratio.
d. Complete the square to find the center and radius of a circle given by an equation.
e. Find an equation for an ellipse given the lengths of its major and minor axes; calculate the area of an ellipse.
f. Use coordinates to solve geometric problems.

   Include proving simple geometric theorems algebraically, using coordinates to compute perimeters and areas for triangles and rectangles, finding midpoints of line segments, finding distances between pairs of points and determining when two lines are parallel or perpendicular.

Vectors and matrices[65]

Core Standards · Students understand that:

1. Vectors are quantities having both magnitude and direction. They are typically represented by directed line segments.
2. On a coordinate plane, vectors are determined by the coordinates of their initial and terminal points or by their x- and y-components.

[65] Limit to vectors in the plane and 2x2 matrices.

3. Vectors can be added end-to-end, component-wise, or by the parallelogram rule. The length of the sum of two vectors is typically not the sum of the lengths.
4. Translations of the plane can be represented by vectors.
5. Vectors are often used to describe "directed quantities" in physics, such as position, velocity, acceleration and force. Vector addition is used to find resultant forces or compute displacements.
6. Multiplying a 2x2 matrix into a vector produces another vector. This can be viewed as a transformation of the plane.
7. A system of two linear equations in two variables can be represented as a single matrix equation in a vector variable.
8. Matrices can be added, subtracted and multiplied.
9. The zero and identify matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a 2x2 matrix determines whether it has a multiplicative inverse.

Core Standards · Students can and do:

a. Represent vectors graphically.
b. Perform basic vector operations (addition, subtraction, scalar multiplication) both graphically and algebraically.
c. Use vectors to model and solve problems.
d. Use trigonometry to decompose a vector into perpendicular components.
e. Add, subtract and multiply matrices.
f. Represent systems of equations as matrix equations.
g. Find the inverse of a matrix if it exists and use it to solve equations.

Complex Numbers

Core Standards · Students understand that:

1. To solve quadratic equations that have no solutions in the real numbers, the number system can be extended to include the square roots of –1, creating a closed number system called the complex numbers.
2. The Properties of Arithmetic and the relation $i^2 = -1$ can be used to perform operations on complex numbers.
3. All polynomials can be factored over the complex numbers, e.g. as in $x^2 + 4 = (x + 2i)(x - 2i)$.
4. Complex numbers can be visualized on the complex plane. Real numbers fall on the horizontal (real) axis, and imaginary numbers fall on the vertical axis.
5. On the complex plane, arithmetic of complex numbers can be interpreted geometrically: addition is analogous to vector addition, and multiplication can be understood as rotation and dilation about the origin. Complex conjugation is reflection across the real axis.
6. The absolute value (or modulus) of a complex number is defined as its distance from the origin in the complex plane. On the complex plane, as on the real line, the distance between numbers is the absolute value of the difference, and the midpoint of a segment is the average of the numbers at its endpoints.
7. Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ links complex numbers to trigonometry.

Core Standards · Students can and do:

a. Add, subtract and multiply complex numbers.
b. Find the conjugate of complex a number and use it to find absolute values and divide complex numbers.
c. Graph complex numbers in both rectangular and polar form and interpret arithmetic of complex numbers geometrically.

d. Solve quadratic equations over the complex numbers.
e. Convert complex numbers between rectangular and polar form.
f. Re-derive trigonometric identities using complex methods.

Mathematics: High School—Modeling

A Coherent Understanding of Modeling.

Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

A model can be very simple, such as a geometric shape to describe a physical object like a coin. Even so simple a model involves making choices. It is up to us whether to model the solid nature of the coin with a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. For some purposes, we might even choose to adjust the right circular cylinder to model more closely the way the coin deviates from the cylinder.

In any given situation, the model we devise depends on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models we can create and analyze is constrained as well by the limitations of our mathematical and technical skills. For example, modeling a physical object, a delivery route, a production schedule, or a comparison of loan amortizations each requires different sets of tools. Networks, spreadsheets and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical structure might model seemingly different situations.

The basic modeling cycle is one of (1) identifying the key features of a situation, (2) creating geometric, algebraic or statistical objects that describe key features of the situation, (3) analyzing and performing operations on these objects to draw conclusions and (4) interpreting the results of the mathematics in terms of the original situation. Choices and assumptions are present throughout this cycle.

The modeling cycle and general tools

Core Standards · Students understand that:

1. The behavior of quantities in physical, economic, public policy, social and everyday situations can be modeled using mathematics. Mathematics is used to model relationships among quantities, constraints and objectives.
2. Models are formulated to answer questions about the world based on an analysis of the situation and a conceptual model that involves assumptions and choices.
3. Quantities in the situation are represented by variables in the model, usually through measurement. Modeling includes decisions about what to measure and how, and how well the measurements work for the purpose.
4. Mathematical knowledge and skill are required in order to get results from a mathematical model—even to devise a model in the first place. Areas of mathematics commonly used in modeling include linear, quadratic, exponential or other functions; probability and statistics; and geometry (solid, plane and coordinate). In active modeling, fluency with math content is required in order to focus on the larger problem.
5. Technology is often required in order to obtain results from a model.
6. The results of a mathematical model must be evaluated against evidence and the phenomena at hand. If the mathematics is correct, then unreasonable results point to unreasonable assumptions and the need to revise the model.
7. Real-world problems do not announce that they are amenable to mathematical analysis and solution; bringing mathematics to bear on such a problem is a highly creative act.

Core Standards · Students can and do:

a. Creatively apply the mathematics they know to situations in which it only imperfectly applies—and achieve useful results by doing so.

For example, independently choose to describe HIV transmission as a random event with a fixed probability per sexual encounter.

b. Decide what measures are relevant to a problem.

For example, given the purpose at hand, is traffic safety best measured in terms of fatalities per year or fatalities per vehicle-mile? (or fatalities per 100 million vehicle-miles?)

c. Use network diagrams or other techniques to visualize complex situations with many factors, causes or agents.

For example, what agents and factors are responsible for setting the price of gasoline? How do they interact?

d. In situations with many factors, causes or agents, organize the factors/causes/agents into a hierarchy of importance.

For example, what are the primary, secondary, and relatively rare causes of lung cancer?

e. Use order of magnitude estimates, unprompted, to identify important effects, disregard unimportant effects and predict results of a more detailed model.

f. Use 2-by-2 tables, flowcharts, and other strategies to organize information and manage scenarios.

Modeling with geometry, equations, functions, probability, and statistics

Core Standards · Students can and do:

a. Model physical objects with geometric shapes.

Include common objects that can reasonably be idealized as two- and three-dimensional geometric shapes. Identify the ways in which the actual shape varies from the idealized geometric model.

b. Model situations with equations and inequalities.

Include situations well described by a linear inequality in two variables or a system of linear inequalities defining a region in the plane.

c. Model situations with common functions.

Include situations well described by linear, quadratic or exponential functions; and situations that can be well described by inverse variation ($f(x) = k/x$). Include identifying a family of functions that models features of a problem, and identifying a particular function of that family and adjusting it to fit by changing parameters. Understand the recursive nature of situations modeled by linear and exponential functions.

d. Model situations using probability and statistics.

Include using simulations to model probabilistic situations; describing the shape of a distribution of values and summarizing a distribution with measures of center and variability; modeling a bivariate relationship using a trend line or a regression line.

Mathematics: High School—Statistics

A Coherent Understanding of Statistics.

[Final draft of CCR narrative goes here.]

Summarizing and interpreting categorical, count and measurement data

Core Standards · Students understand that:

1. Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.
2. Visual displays and summary statistics condense the information in data sets into usable knowledge.

Core Standards · Students can and do:

a. Summarize comparative or bivariate categorical data in two-way frequency tables; interpret joint, marginal and conditional relative frequencies in the context of the data.
b. Compare data on two or more count or measurement variables by using plots on the real number line (dot plots, histograms and box plots); use appropriate statistics to summarize center (median, mean) and spread (interquartile range, standard deviation) of the data sets; interpret changes in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points.
c. Summarize bivariate quantitative data by giving a regression line and a measure of goodness of fit.

Making inferences and justifying conclusions drawn from data

Core Standards · Students understand that:

1. Statistics is a process for making inferences about population parameters based on a sample from that population; randomness is the foundation for statistical inference.
2. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

Core Standards · Students can and do:

a. Use probabilistic reasoning to decide if a specified model is consistent with a given data-generating process.
b. Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.
c. Use data from a sample survey to estimate a population parameter.
d. Use data from a randomized experiment to compare two treatments.
e. Evaluate reports based on data.

Mathematics: High School—Probability

A Coherent Understanding of Probability.

[Final draft of CCR narrative goes here.]

Modeling random events with finite sample spaces

Core Standards · Students understand that:

1. Random phenomena can be modeled mathematically using a sample space in which sample points represent distinct outcomes, and in which each sample point is assumed to have the same probability.
2. Events are subsets of a sample space that can be defined using characteristics (or categories) of the sample points, as well as unions, intersections, or complements thereof ('and', 'or', 'not'). A sample point may belong to several events (categories).
3. If A and B are two events (categories), then the conditional probability of A given B, denoted by $p(A \mid B)$, is the fraction of sample points in B that also lie in A.
4. The laws of probability can be used to generate new probabilities from known probabilities.

Core Standards · Students can and do:

a. Compute theoretical probabilities of compound events by constructing and analyzing representations, including tree diagrams, systematic lists, and Venn diagrams
b. Use the addition and multiplication laws of probability to compute probabilities of complementary, disjunctive, and compound events.
c. Apply concepts such as intersections, unions and complements of events, and conditional probability and independence, to define or analyze compound events, calculate probabilities, and solve problems.
d. Construct and interpret two way tables to show probabilities when two characteristics (or categories) are associated with each sample point. Use a two way table to determine conditional probabilities.
e. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
f. Use permutations and combinations to compute probabilities of compound events and solve problems.

Experimenting and simulating to model probabilities

Core Standards · Students understand that:

1. Sets of data obtained from surveys, simulations, or other means, can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data. The probability of an event within the data set is its relative frequency.
2. The law of large numbers provides the basis for estimating certain probabilities by use of empirical relative frequencies.
3. The probability of an outcome can be interpreted as an assertion about the long-run proportion of the time the outcome will occur if the random experiment is repeated a large number of times. The observed proportion of occurrence for the outcome of interest can be used as an estimate of the relevant probability.

Core Standards · Students can and do:

a. Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

b. Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities. When there are substantial discrepancies between predicted and observed probabilities, explain them in terms of the assumptions of the probability model.
c. Use the relationship between conditional probabilities and relative frequencies in contingency tables to analyze decision problems.
d. Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.
e. Apply the binomial theorem to solve probability problems.

Using probability to make decisions

Core Standards · Students understand that:

1. A probability distribution is a collection of probabilities $\{p_1, \ldots, p_n\}$ for a set of mutually exclusive and jointly exhaustive events $\{E_1, \ldots, E_n\}$. The probabilities in a probability distribution sum to unity.
2. A random variable attaches a value to each event in a probability distribution. The expected value of the random variable is the weighted average of its possible values, with weights given by their respective probabilities.
3. When the possible outcomes of a decision can be assigned probabilities and payoff values, the decision can be analyzed as a random variable with an expected value, e.g. of a wager. If possible, this is the first thing to compute in a decision context.

Core Standards · Students can and do:

a. Calculate expected value to analyze mathematical fairness, payoff.
b. Evaluate and compare options in situations where all of the available options share the same expected value but carry different levels of risk.
c. Analyze each of two options and make a quantitatively informed decision in situations where one option has both a higher expected return and a higher level of risk. Include both low-stakes and high-stakes decisions.
d. Analyze decision problems using probability concepts.

Mathematics: High School—Geometry

A Coherent Understanding of Geometry.

[Final draft of CCR narrative goes here.]

Triangle Congruence

Core Standards · Students understand that:

1. Rigid motions move lines to lines and segments to segments; preserve the distance between points; and preserve measures of angles.
2. Two geometric figures are congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition.
3. Criteria for triangle congruence can be thought of as answers to the following question: What information about the measures in a triangle ensures that all triangles drawn with those measures are congruent?
4. Criteria for triangle congruence can be established using rigid motions.

Core Standards · Students can and do:

a. Use (in reasoning and problem solving) precise definitions of angles, polygons, parallel and perpendicular lines, rigid motions (rotations, reflections, translations), parallelograms and rectangles; commit these definitions to memory.
b. Prove theorems about lines and angles; test conjectures and identify logical errors in fallacious proofs.

Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; measures of supplementary angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints.

c. Prove theorems about triangles; test conjectures and identify logical errors in fallacious proofs.

Theorems include: measures of interior angles of a triangle sum to 180°, base angles of isosceles triangles are equal, the triangle inequality, the longest side of a triangle faces the largest side and vice-versa, the exterior-angle inequality, and the segment joining midpoints of two sides of a triangle parallel to the third side and half the length.

d. Use and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid and kite.
e. Characterize parallelograms in terms of equality of opposite sides, in terms of equality of opposite angles, and in terms of bisection of diagonals; characterize rectangles as parallelograms with equal diagonals.

Similarity, Right Triangles and Trigonometry

Core Standards · Students understand that:

1. The dilation of a given line is parallel to the given line. (In particular, lines passing through the center remain unchanged.)
2. The dilation of a given segment is parallel to the given segment and longer or shorter in the ratio given by the scale factor. A dilation leaves a segment unchanged if and only if the scale factor is 1.
3. The assumed properties of dilations can be used to establish the AA, SAS and SSS criteria for similarity of triangles.
4. Similarity allows one to view side ratios in right triangles as properties of the angles themselves, leading to elementary definitions of sine, cosine and tangent.

Core Standards · Students can and do:

a. Use triangle similarity criteria to solve problems and to prove relationships in geometric figures.
b. Prove that two lines with well-defined slopes are parallel if and only if they have the same slope, and perpendicular if and only if the product of their slopes is equal to –1.
c. Give an informal explanation using successive approximation that a dilation of scale factor r changes the length of a curve by a factor of r and the area of a region by a factor of r^2.
d. Use and explain the relationship between the trigonometric ratios of complementary angles.
e. Use trigonometric ratios and the Pythagorean theorem to solve right triangles[66] in applied problems.

Circles

Core Standards · Students understand that:

1. All circles are similar.
2. There is a unique circle through three non-collinear points, or tangent to three non-concurrent lines.

Core Standards · Students can and do:

a. Identify and describe relationships among angles, radii, and chords.

Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

b. Identify and define radius, diameter, chord, tangent, secant and circumference.
c. Determine the arc lengths and the areas of sectors of circles, using proportions.
d. Construct a tangent line from a point outside a given circle to the circle.
e. Prove and use basic theorems about circles, and use these theorems to solve problems. Include:

- Symmetries of a circle
- Similarity of a circle to any other
- Tangent line, perpendicularity to a radius
- Inscribed angles in a circle, relationship to central angles, and equality of inscribed angles
- Properties of chords, tangents and secants as an application of triangle similarity.

Axiomatic Systems

Core Standards · Students understand that:

1. Mathematical statements are proven or disproven by deductive reasoning. Conjectures can arise from inductive reasoning, but they cannot be proven that way.
2. Precise definitions make possible rigorous logical reasoning, and definitions shared in common make possible the objective evaluation of one's own reasoning by others.
3. Logical reasoning requires avoiding common fallacies, such as using an example to prove the rule or confusing a statement with its converse.
4. Axiomatic systems require precise definitions, but some terms must be left "undefined." The axioms specify how the undefined terms behave.
5. The first three postulates of the *Elements* are models of straightedge and compass construction.
6. Hilbert and other mathematicians improved on the *Elements* by identifying its hidden assumptions and making them explicit with additional axioms.

[66] A right triangle has five parameters, its three lengths and two acute angles. Given a length and any other parameter, "solving a right triangle" means finding the remaining three parameters. (It is worth reflecting on why this problem is well-posed.)

7. Three classical construction problems (trisecting an angle, duplicating a cube and squaring a circle) inspired the development of much important mathematics.

8. The Parallel Postulate (axiom) distinguishes Euclidean geometry from other geometries. Other geometries, such as spherical and hyperbolic geometry, use alternatives to the Parallel Postulate. Many theorems of Euclidean geometry are not theorems in other geometries.

Core Standards · Students can and do:

a. Use the terms point, line and plane to define other geometric terms as line segments, angles and rays.
b. With ruler and compass:
   - Divide a segment into any number of equal parts.
   - Given two segments of lengths r and s, construct a segment of length rs and one of length r/s.
   - Given a segment of length r, construct a segment of length $\sqrt{r}$.

Trigonometry of General Triangles

Core Standards · Students understand that:

1. The Law of Sines generalizes the side-angle inequality.

2. The Law of Cosines generalizes the Pythagorean theorem.

3. The Laws of Sines and Cosines embody the triangle congruence criteria, in that three pieces of information are usually sufficient to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that "Side-Side-Angle" is not a congruence criterion.

Core Standards · Students can and do:

a. Explain proofs of the Law of Sines and the Law of Cosines.
b. Use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Geometric Measurement and Dimension

Core Standards · Students understand that:

1. Congruence plays a fundamental role defining the concepts of length, area and volume.

2. Areas of polygons can be computed by dissecting them into triangles and using the fundamental property of area, that the area of a dissected figure is the sum of the areas of its components.

3. Lengths of curves and areas of curved regions can be defined informally using the concept of "limit."

4. Cavalieri's principle allows one to understand volume formulas informally by visualizing volumes as stacks of thin slices.

Core Standards · Students can and do:

a. Give definitions of rectangular prism, (right) pyramid, (right circular) cone, (right circular) cylinder and sphere.

b. For a pyramid or a cone, give an heuristic argument to show why its volume is (1/3) its height times the area of its base.

c. Use the behavior of length and area under dilations to prove the formulas for the circumference and area of a circle.

d. Apply formulas and solve problems involving volume and surface area of right prisms, right circular cylinders and right pyramids, cones, spheres and composite figures.

e. Identify and apply the 3:2:1 relationship among volumes of circular cylinders, hemispheres and cones with same height and circular base and 3:1 relationship between volume of a prism and pyramid with same base area and height.

f. Identify cross-sectional shapes of slices of three-dimensional objects, and identify three-dimensional objects traced out by rotations of two-dimensional objects.

Mathematics: High School—Calculus

Calculus is an important part of the high school curriculum for a large and growing number of students. To see well-established standards for this course, please see course descriptions such as those of the College Board, International Baccalaureate Organization, or any of the following states: California, Florida, Hawaii, Indiana, Mississippi, Pennsylvania, South Carolina, Tennessee, Utah, and Virginia. We invite feedback from states as to whether they would like to see Calculus in future drafts of the Common Core Standards.

Progressions in Grades K–8

Note, a progression may appear in more than one band.

Grades K-5

Number

Counting and Cardinality
Base Ten Computation
Early Relations and Operations
Quantity and Measurement
Operations and the Problems They Solve
Fractions

Geometry

Shapes
Coordinates
Geometry

Data

Statistics

Grades 6-8

Number

The Number System

Algebra

Ratios and Proportional Relationships
Expressions and Equations
Functions and the Situations They Model

Geometry

Geometry

Data

Statistics
Probability

List of Progressions and Grade Ranges

Strand	Progression	Start	End
Number	Counting and Cardinality	K	K
	Early Relations and Operations	K	1
	Base Ten Computation	K	5
	Quantity and Measurement	K	5
	Operations and the Problems They Solve	2	4
	Fractions	3	5
	Ratios and Proportional Relationships	6	7
	The Number System	6	8
Geometry	Shapes	K	4
	Coordinate Geometry	5	5
	Geometry	6	8
Algebra*	Expressions and Equations	6	7
	Functions & The Situations They Model	8	8
Data*	Statistics	5	8
	Probability	7	8

* The Algebra and Data strands have concepts and skills in earlier grades that progressions in the Number strand.

GLOSSARY

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $3/4$ and $-3/4$ are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.

Algorithm. A step by step routine that always gives some answer, rather than ever giving no answer; that always gives the right answer, and never gives a wrong answer; that can always be completed in a finite number of steps, rather than in an infinite number of steps; and that applies to all problems of a given type (e.g., adding any two multidigit whole numbers, or bisecting any angle). Cf. Wikipedia's "effective procedure," from which this definition is adapted.

Common logarithm. The common logarithm of x is the power to which you raise 10 in order to get x.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by a sequence of rigid motions (rotations, reflections, and translations).

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Integer. A positive whole number, a negative whole number, or 0.

Mean. The sum of the values in a list divided by the number of values in the list. (To be more precise, this defines the *arithmetic mean*.)

Median. In a list of values, the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $3/4$ and $4/3$ are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

Range. The difference between the greatest and smallest values in a list of numbers.

Rational number. A number expressible in the form a/b for integers a and $b \neq 0$. The rational numbers include positive and negative integers, positive and negative fractions, and 0.

Related fractions. Two fractions are *related* if one denominator is a factor of the other. (See Ginsburg, Leinwand and Decker (2009), *Informing Grades 1-6 Mathematics Standards Development: What Can Be Learned from High-Performing Hong Kong, Korea, and Singapore?*, Table A1, p. A-5, grades 3 and 4.)

Similarity transformation. A rigid motion followed by a dilation.

Single-place number. The numbers that result when a whole number between 1 and 9 (inclusive) is multiplied by the numbers 10, 100, 1000, etc.

Teen number. A whole number that is greater than or equal to 11 and less than or equal to 19.

Transitive property of measurement order. If one object is bigger than a second, and the second object is bigger than a third object, then the first object is bigger than the third object.

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International Benchmarking and the Common Core

The Common Core State Standards (CCSS) are designed to be **college- and career-ready** and **internationally benchmarked**. To that end, the development process included the review and consideration of many sources, including research studies, existing standards from the U.S and abroad, and the professional judgment of teachers, content area experts, and college faculty. This paper will briefly describe how international benchmarking was used to develop the CCSS.

What documents were used to ensure that the CCSS were internationally benchmarked?

To ensure that the standards prepare students to be globally competitive, the development team used a number of sources, including: the frameworks for PISA and TIMSS; the International Baccalaureate syllabi; the American Institutes for Research report , *Informing Grades 1-6 Mathematics Standards Development: What Can Be Learned From High-Performing Hong Kong, Korea, and Singapore* and; the A+ Composite found in *A Coherent Curriculum: The Case for Mathematics* by Bill Schmidt, Richard Houang, and Leland Cogan.

In addition, the development team looked to the standards of a number of individual countries and provinces to inform the content, structure and language of the CCSS. In *mathematics*, twelve set of standards were selected to help guide the writing of the standards: Belgium, Canada [Alberta], China, Chinese Taipei, England, Finland, Hong Kong, India, Ireland, Japan, Korea, and Singapore.[i] In *English language arts*, the writing team looked closely at ten sets of standards from Australia (New South Wales and Victoria), Canada (Alberta, British Columbia, and Ontario), England, Finland, Hong Kong, Ireland, and Singapore.[ii]

How were the international benchmarks used to inform the development of the CCSS?

The goal of the international benchmarking in the common core state standards development process was to ensure that the CCSS are as rigorous as comparable standards in the high-performing and other countries. However, the use of international benchmarks as evidence is no easy feat; it is not simply a matter of identifying the "best" source and copying it, or of aggregating all viable sources to find some set of shared expectations. Rather, international benchmarks were used to guide critical decisions in the following areas:

- *Whether particular content should be included:* One of the principal ways international standards were used in this development process was as a guide when making tough decisions about whether content should be included or excluded.
- *When content should be introduced and how that content should progress*: The progression of topics in the international mathematics standards helped the development team make decisions about when to introduce topics in the CCSS as well as when to stop focusing on them.
- *Ensuring focus and coherence*: Standards from other countries tend to be very focused, including only what is absolutely necessary.
- *Organizing and formatting the standards*: Certain organizational aspects or characteristics of international standards that promoted clarity and ease of reading and use served as a model for the CCSS.
- *Determining emphasis on particular topics in standards*: Where emphasis on particular topics was found repeatedly in international standard, this was instructive in determining their importance for inclusion in the CCSS.

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When the final version of the K-12 Common Core State Standards is released, it will be accompanied by a discussion of the evidence that was used in their development. In the meantime, the evidence from the September 2009 draft of the College and Career Ready Standards is available: The URL for the ELA document is http://www.corestandards.org/Files/ELAEvidence.pdf, and the URL for the mathematics document is http://www.corestandards.org/Files/MathEvidence.pdf.

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[i] Eight of these were high-performers on either TIMSS, PISA or both: Belgium, Canada [Alberta], Chinese Taipei, Finland, Hong Kong, Japan, Korea, and Singapore. England and Ireland, which have uneven performances on international assessments, were included because of their cultural links to the United States. China and India were included because of their growing global competitiveness.

[ii] Differences in language have a greater impact on the teaching and learning of language arts than of mathematics, so the teams looked primarily at English-speaking countries. All were high-performers on PISA except Singapore, which did not participate, and England, which as in mathematics was selected partly for its cultural links to the United States.