



"Bringing the Common Core to Life"
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Chancellors Hall - State Education Building - Albany, NY
April 28, 2011

Part 8
Discussion of the Common Core State
Standards for Mathematics
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This is a bit of a shift to mathematics. Those of you who would like to leave the room can do so at this point. In terms of getting more concrete in mathematics, I want to try to answer two to three questions that you should have in your mind as we talk about it.

One is you've talked a lot about focus and depth and richness. What then goes away? What does that do in assessment terms to speak very bluntly about it? And what's the difference between that kind of instruction and the kind of instruction we're doing now? So this is a picture as you can see in front of you of New York State 2005 Mathematics Standards. There's a lot to recommend it. One thing that's very good about it and early in its work is the nice integration of the kind of practices or process standards with the content strands that go vertically. That's something really nice about it. However, there is one way where it echoes the great American habit in mathematics which if we go to the next slide it will become clear, which is what I like to call the shopping-aisle approach to populating mathematics instruction in K-12. Do these strands look familiar to everybody? No matter what grade you're in, they're the same, right? And they're like shopping aisles. That is, for every test and for every curriculum you need to fill them in. You need to have stuff in numbers sense and operations, algebra, geometry, measurement, statistics, and probability. That section is to teach standards. So we add standards there and we make the test bigger. You've got to shop in every aisle. Because we are kind of even minded in America, we shop pretty equally in each strand, right? Have you noticed that? You've got the data people working on the data and they're like, "Go data!" And then the geometry people and they're like, "Go geometry!" And then you get the pre-algebra people, and then each of them fights for their stuff getting in. The only problem here is this has no relationship to the true balance of what mathematics and early mathematics in particular is most important. Remember, it was only three topics that marked the high-performing countries in mathematics. And in the core standards they coalesce around K-2 addition and subtraction of whole numbers and the quantities they measure, the units that they measure, moving into multiplication and division and fractions in 3-5, obviously very heavy in number sense and operations. The dominant craft in arithmetic lies mostly here. If you look at algebra, it's very interesting. What does pre-algebra mean? In America, what pre-algebra has come to mean is patterns and patterns are the one area in which America



leads the world in mathematics. Patterns problems are problems of this sort: you have three types of cheese, two types of meat, four types of bread, how many sandwiches can you make? I am not saying here, please, that mathematics is not in some deep sense about patterning but I am saying the precursor to algebra is not practicing patterns problems, it is what--fractions. If I say $3x = 6$, I hope a fraction is beginning to form in your mind. That command of number is essential for later mathematics performance. In geometry terms, key will actually be at the intersection of both geometry and measurement. That is precisely not geometry apart from measurement, just knowing this is a trapezoid, but a rectangle as an expression of multiplication and a way to understand units is essential to the command of number. So it's actually not as separate strands best understood but their interaction in K-8 that lends the most powerful support to the core things you're learning.

Finally, statistics and probability I will say unromantically about K-5 mostly just weight. Some sampling here but while it is adorable and wonderful that we have kids making little bar charts all the time and counting things, it is fairly low-level math. And it's evading the more difficult work of number and manipulation and they can do much more data work in middle school when there are vast numbers and amounts of data. And guess what? A lot of that work should be happening in science classrooms where there actually is data and dense data to look at. So we are willing to make hard choices here to focus on the math that matters the most and really build a staircase to college and career. My terrific colleague Sandra Alberti put this to me once and I think it's absolutely true. There's a practice in math we all have to admit, which is when it gets too hard move on to the next topic. This is the game. And because the assessment measures everything, it's an equal chance – they may test this, they may test that -- why not move on to the next topic? To me, the great example of this is the time-honored time and money problem. So much time spent in the early grades on time and money problems. Let me present for your consideration two children, one who is expert in time and money problems and one who is expert in fractions and operations. Who do you think is likely more ready for algebra? No question, right? Because time and money are but a single application of a much broader and more varied set of skills. I am not saying, lest the emails pour in, that you should not mention or do some work on time and money. But let's be honest about how dominant it is and how unrelated it is, that obsessiveness to the core developed in mathematics and flexibility students really need.

Let's keep going. Let me show you a different picture. If you look at this picture of the growth, notice now not all strands are equal on the actual development of algebra, but three things become absolutely primary and powerful in driving that train in K-5--operations and algebraic thinking. By this I mean, what you learn about the operations that persist like the role of subtraction, like the commutative property, like other properties of operations that you learn that will help you when doing guess what--equations and expressions--where those same laws hold true. Number and Operations—Base Ten we've talked a lot



about whole numbers here, a lot about their manipulation and that gives you one part of the number system you're going to develop. But crucially, the number and operations surrounding fractions will then begin to fill out the number line and of course the great excitement of 6th through 8th grade is moving into a numbers system that now includes negative numbers in 6th grade. But you see how these flow. You're constantly reusing the same concepts in the growth of the staircase, leading to an algebraic ways of thinking that you begin to master linear algebra in grade 8 and go on to a wider set of algebra in the high school. This is not a complete picture of mathematics. Crucially, in middle school, we're going to do a lot of work on proportional reasoning. We're going to do a lot of work on geometric measurement like I was talking about, so much power in the world to think proportionately leading to functions, of course. In geometric measurement, so much of engineering and also of life is to be able to see how to measure shapes. But, do you see how exciting it is that no longer do we have to act like every strand is equal? We can rather focus on what's most important and let that give us a much more compelling picture of mathematics that's actually descriptive and meaningful, that shows teachers and kids a way from here to there because that's what standards must be at their mightiest which is a clear path to where we're going, not a confusing bureaucratic system.

If we go on now, I would like to get a little more specific by comparing a New York State Standard in fractions to the core standards in math. The second one is unreadable for most of you in the room which helps me because you can't really check me but I'll walk you through it in some detail. The 6th grade New York State Standard is multiply and divide fractions with unlike denominators. Now you may remember when I talked to you earlier I said one of those mysterious things about American math education is none of us really know what the heck we're doing when we divide a fraction. Let me begin to explain the mystery of why does that happen because this is the first time in the New York State Standards where the multiplication and division of fractions is mentioned is in grade 6. It emerges suddenly. It is a very clear standard. It is a very important standard, but emerges utterly without earlier focus on multiplication and division of fractions. In my mind it's kind of like an immaculate conception. It's kind of something that has come out of nothing. While there may be a big role for such things, it's probably not in mathematics. What happens here is we're actually asking kids to make a huge leap here. This is a major advance and what the core standards do – I really want to get over the idea that the core standards are just harder -- they are harder and more demanding but they try to build that through an elegance and simplicity and more time to practice. You saw it in the reading example.

There is something simple about getting rid of a bunch of the other stuff and focusing on reading. Just like that in math, let's look at what kids are now doing in 4th grade to prepare for that multiplication and division of fractions. The standard reads the following: "Apply and extend previous understandings of multiplication" so building on our understanding of multiplication, we're going to extend it to multiplying

first a fraction by a whole number. This whole standard is just about multiplying fractions by whole numbers. It's the simplest case of multiplication of fractions. In 5th grade, we're going to get to multiplying fractions by fractions but first we're going to master multiplication of a fraction by a whole number. By commanding the simplest case, you create that staircase to further master it. The first point it makes is "Understand a fraction a/b as a multiple of $1/b$." So it's saying that I have to understand to fulfill the first standard that $3/4$ is $3 \times 1/4$. Who cares? Why do I have to understand anything else? I've got one fraction $3/4$ and I've got another standard $1/4$. Could these math people leave us alone? But if I just understand them as two numbers in that way, what if I have 4 different parts to a car? Are 3 of them $3/4$? No, because they're not the same, right? It has to be 3 times the same 1 to get the notion of fraction that's operating here. Be very careful. So in understanding fractions use a visual fraction model to represent $5/4$ as 5 products of the same $1/4$. Look what's also happened. All of a sudden multiplication and fractions are not different. Hiding within $3/4$ is $3 \times 1/4$. To even understand $3/4$, you've got to get enough multiplication to grasp $3 \times 1/4$. Multiplying fractions has all of a sudden become part of understanding them to begin with. It's not a big surprise that we're suddenly going to move to multiplying them by others--does that make sense?--because it's part of your original understanding of them. Again, "b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number." And then it says, "For example, use a visual fraction model to express" this. Finally, it requires that you "Solve word problems" doing this. Look at the attention lavished on this fundamental capacity to do it visually and through word problems and what that gets at is an essential part of the math standards which are the practices. So as you know at the beginning of the math standards there's a set of essential practices that are meant to guide instruction in math, things like kids seeing the structure of the work they're doing as well as moving to solve things, an attempt to celebrate precision and accuracy. In these math standards let me warn you that estimation is the product of several attempts at precision and accuracy rather than the avoidance of it. That is, these math standards do celebrate in addition to word problems and deep understanding, fluency, accuracy, and speed. They can both go together when you do a few things well. What you see here is the practices are not a separate body of material but woven into your understanding of fractions, the ability to visualize and see what you're doing, your ability at the same time to solve it in diverse situations including word problems.

This is not super complicated stuff in that it's just whole numbers times fractions. But by doing it with this kind of care, you're getting ready for that really demanding 6th grade move, which is to multiply and divide them with confidence. Is that making sense that it's that kind of slowness and care? If you were going to look at one thing in the core standards after this, I would really invite you to look at the progression of fractions. I think they are quite beautiful actually to look at how fractions unfold from 3rd grade to 6th grade and the kind of care. You'll notice that the definition of multiplication that actually began earlier in 3rd grade is consistent here. That's what I mean by coherence in mathematics. Everything you learn builds.

It's not like you waste it, like wasting a ton of time on patterns problems that you don't reuse or a ton of time doing all this time on money stuff that you don't really use again or a ton of data stuff that isn't quite reusable at higher levels. You instead focus on the powerful stuff, which is difficult and demanding and requires patience and practice but it's constantly building.

I think the underlying ethics of this are very clear to me. Someone in Louisiana was talking to me about this and he is a psychometrician by training, he's not a mathematician, but a great example of another career that relies all the time on math. He said to me, "You know, there are people who always say that there are math people and there are not math people." Along with their foolish left brain, right brain people friends which I won't even talk about. Let's just say this, what if that is all an illusion? What if there were no such thing as math people or not math people? But there is a group of people that had enough practice with the core of number and operations and the command of it and the quantities that measure and those other things supporting that mastery, enough practice so that door opens? If that door does not open, none of math opens to you. I want to be very clear. I've been hopeful in this conversation, but I want to be equally clear that if you don't adequately practice this core arithmetic and master it, all of later math is beyond you. So it is at the same time urgent and needed but that is what we call a mathematical person. It is making concrete what we talk about.

If we go on for a minute I'll show you briefly what this means about assessment and instruction. So it seems that in key moments in the curriculum, much like in reading, we slow down and that the mile-wide inch-deep curriculum is replaced by a set of intense engagements around things that matter. Let's go on for a minute. This is today's version of the world with the shopping aisles – I mean those colors to represent the existing 5 strands. While number and operations is kind of the biggest thing, you kind of have to do everything. That's basically the measure we keep giving, the kind of do everything, try to do it well, good luck. Imagine another picture which is where we have 70% of our work in areas of intensive focus that pay off remarkably later where we demand and require real competency on the part of students. This is a high level of demand around fewer things. In the middle are things that we rethink and link, remember like geometry and measurement? You don't just teach them like we always do, you teach them in the ways that they most powerfully contribute to the core commands you need. Finally, there are just things that we do just sample and do less of. Some awareness of them, some presence of them on our assessment, but they're not our focus. If you turn the page to the final one, you'll see we dared to get quite concrete about what the core standards look like today in terms of that. So in K-2 you'll notice the addition and subtraction concepts, skills and problem solving is at the utter intense core. Geometry and measurement we're rethinking so it supports that core. At the same time, we're reducing evidence on patterns, probability, and on estimating because remember estimating is not a good precise thing in and of itself but the product of several attempts at being precise and excellent. And then you gain the skill of



estimation because you know your way around and you see what's roughly like something. But we've made it instead a separate body of expertise and you can see it pervade the rest of the grades.

I want to pause in a minute but leave you with a few thoughts about what I think this means for you in terms of math instruction. What would I urge you to do? Number One: Dare to focus immediately on what matters most in mathematics. It will take time. I believe if I understand the state's plans correctly, the assessment will not change this year but it will change the following year. It will approach more closely, we're still working it out, places where these intensive focuses are much more clear and which really aligns with the common core.

New York is very lucky. Through really courageous assessment leadership, it is making an early transition to the demands of this core before the exams in 2014-2015. So that's a wonderful advantage this state has in delivering that early focus but as one man I would advise you to begin focusing as early as possible. Why? I'll tell you a story about Hong Kong. In the international TIMSS study I referenced earlier, Hong Kong has within its curriculum half the subjects studied on TIMSS. We the Americans have them all. Guess who performs better on the tests? Why? Because those core things are so powerful and flexible, you can apply them to novel situations you haven't necessarily seen before. That's at the heartbeat of this. The flex of the masteries allows kids to be more ready. So it's no longer a question of what's on the test. It's what are you ready for? This test might be different from that test but what we're really interested here is the rough and readiness that prevents you from getting ripped off by people, that let's you use math when you're not asked to do so, and at the same time allows you to compute precisely and effectively when asked to do so. As you look forward to the future of assessment, places where David Steiner and John and others are going to be very influential, we're trying to build an assessment system that rewards this range of math that pays attention to fluency, that pays attention to deep application and modeling, that honors this core work that teachers can do.

I would secondly say to teachers it is totally fine to learn this math as you go. None of us have the deep understanding of elementary mathematics. There're very few of us that this mathematics require. I ask you as teachers to enjoy this chance over the next couple of years, to get into it, to spend more time doing it, to practice your own fluency and understanding, to play around with it. It's OK. It'll already help your students just by focusing more on what matters most. You don't have to be expert immediately. You along with your students will become more and more expert as teachers across the world do by focusing on a few things that they learned to do better and better but only gradually and that's perfectly fine.



I will conclude by saying in all frankness that while they may never invite me again I would say that I have no more confidence in any state in the union of teams that I've visited than the team here in New York State and terrifically with their partners in the union who are delighted about this work and doing everything they can to further it in making this happen. This is not a simple road, but there is a lot of good support for it. Thank you very much.